

Katie was just summoned for jury duty

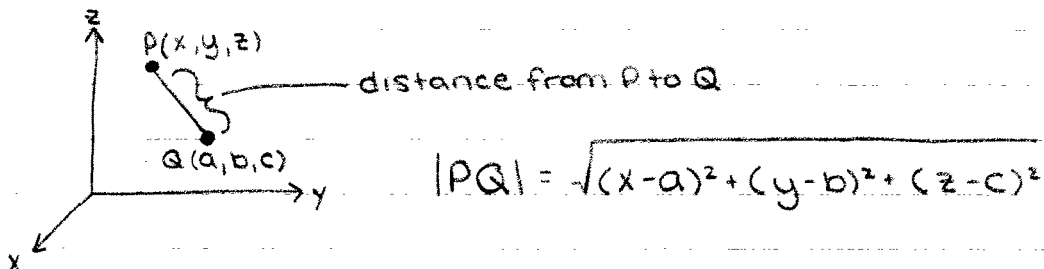
9/23

Outline:

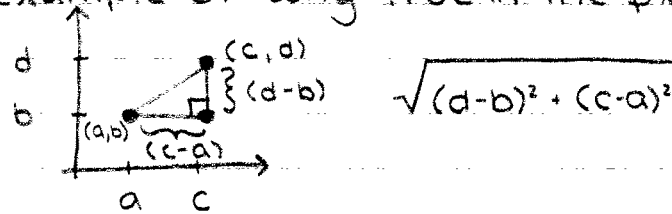
- 3D Stuff
- Vectors

Announcements:

- HW: m. 9/14, w. 9/16, Th 9/17
- Exam coming up



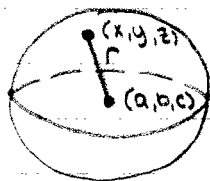
example of why true in the plane:



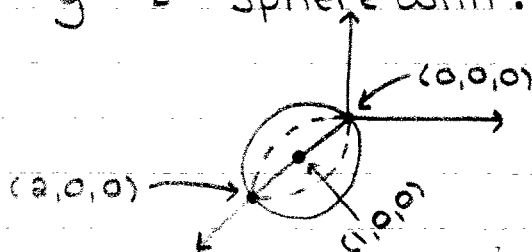
Sphere: The set of all points an equal distance away from a given point

$$r = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} \rightarrow r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$$

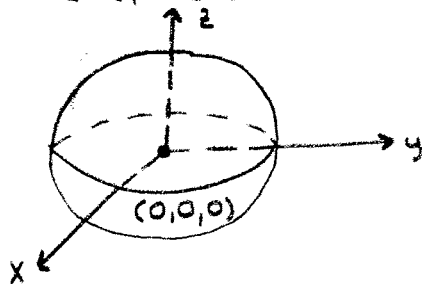
equation for a sphere of radius r with center (a, b, c)



$1 = (x-1)^2 + y^2 + z^2$ Sphere with: radius = 1 center $(1, 0, 0)$

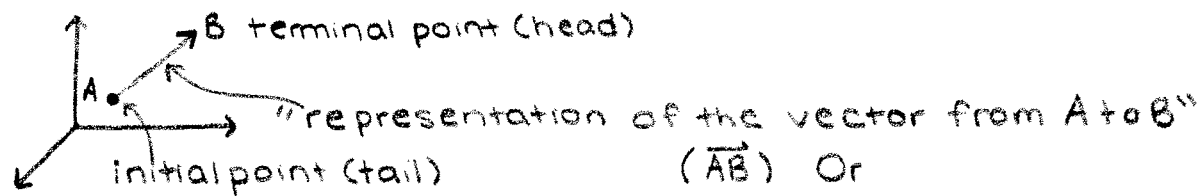


"Advice": draw the sphere first and then the axes



Vectors

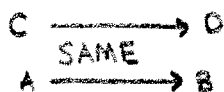
A vector is magnitude & a direction



(\vec{AB}) Or

"the displacement vector from A to B"

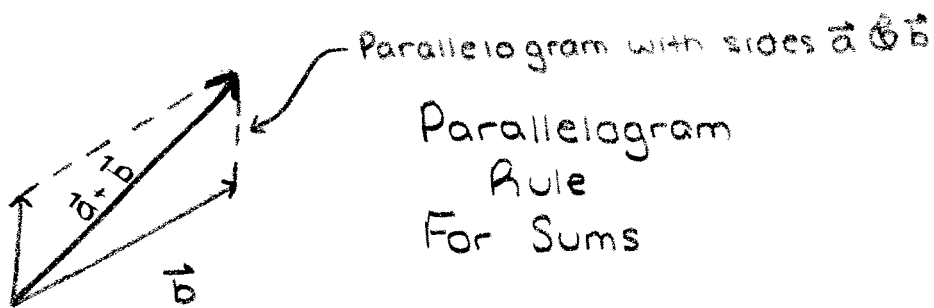
Two vectors \vec{CD} and \vec{AB} are equal if they point in the same direction & have the same length.



(Almost) always we draw vectors in a plane

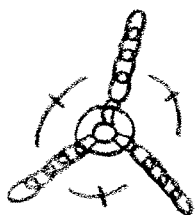
Vector Addition

new notation \vec{a}
(in book **a**)



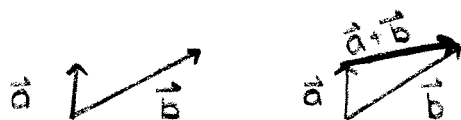
Parallelogram
Rule
For Sums

Example:



all the same length chains
if 3 people pull with the same
force, the ring goes nowhere

Another way to think of vector sums:

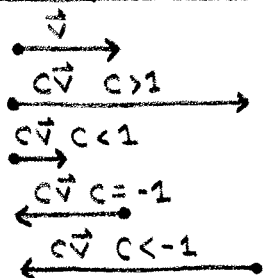


Triangle Rule
For Sums

"Scalar" multiplication

(Scalar means "number")

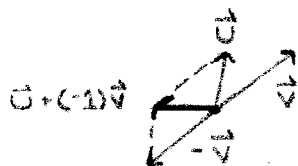
(c is the scalar)



the vector $c\vec{v}$ points in the same or opposite direction as \vec{v} and its length is $|c| \cdot \text{length } \vec{v}$

"is defined to be"

$$\vec{u} - \vec{v} := \vec{u} + (-1)\vec{v}$$



or $\vec{u} - \vec{v}$ is the vector from the head of \vec{v} to the head of \vec{u}

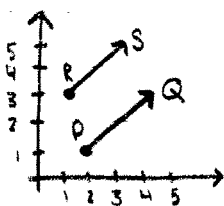


Vectors using components

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \quad \vec{b} = \langle b_1, b_2, b_3 \rangle$$

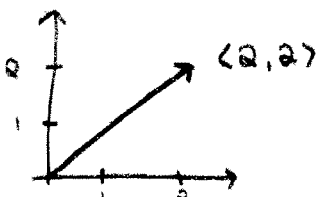
(2d-vectors $\vec{a} = \langle a_1, a_2 \rangle$ & $\vec{b} = \langle b_1, b_2 \rangle$)

Example in the plane:



$$\vec{PQ} = \langle 4-2, 3-1 \rangle = \langle 2, 2 \rangle$$

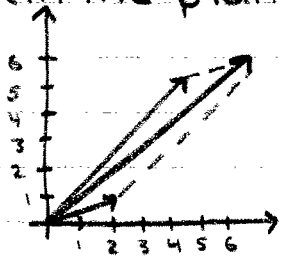
$$\vec{RS} = \langle 3-1, 5-3 \rangle = \langle 2, 2 \rangle$$



When we use components our vectors are based at the origin

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

Example (in the plane):



$$\langle 2, 1 \rangle + \langle 4, 5 \rangle = \langle 6, 6 \rangle$$

length of a vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\text{is } |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$c\vec{v} = \langle cv_1, cv_2, cv_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

Example:

$$\vec{a} = \langle 1, 0, 3 \rangle \quad \vec{b} = \langle -2, -1, 7 \rangle$$

$$2\vec{a} + 3\vec{b} = 2\langle 1, 0, 3 \rangle + 3\langle -2, -1, 7 \rangle$$

$$= \langle 2, 0, 6 \rangle + \langle -6, -3, 21 \rangle$$

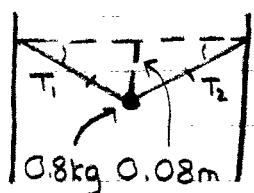
$$= \langle -4, -3, 27 \rangle$$

$$|\vec{a}| = \sqrt{1^2 + 0^2 + 3^2} = \sqrt{10}$$

Exercise: Find the big box of laws/rules for vectors (chapter 13.2) and know how to use them

Example Problem Chapter 13.2 #33

A clothesline is tied between two poles, 8m apart. The line is quite taut and has negligible sag. When a wet shirt with a mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8cm. Find the tension in each half of the clothesline.



$$0.8 \text{ kg} \times 9.8 \text{ m/s}^2 = 7.84 \text{ N}$$

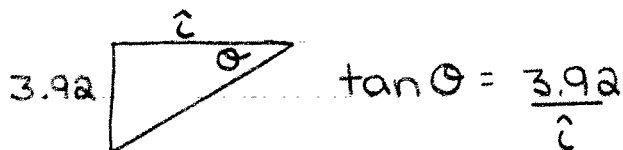
$$T_1 \hat{j} + T_2 \hat{j} = 7.84 \hat{j} \quad \text{and} \quad T_1 \hat{j} = T_2 \hat{j}$$

$$\therefore 2T \hat{j} = 7.84 \hat{j}$$

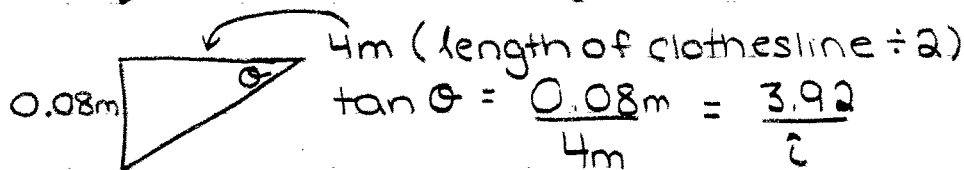
$$T \hat{j} = 3.92 \hat{j}$$

$$T_1 \hat{i} + T_2 \hat{i} = 0$$

$$\therefore T_2 \hat{i} = -T_1 \hat{i}$$



$$\tan \theta = \frac{3.92}{\hat{i}}$$



$$\tan \theta = \frac{0.08 \text{ m}}{4 \text{ m}} = \frac{3.92}{\hat{i}}$$

$$0.08 \hat{i} = 3.92(4)$$

$$\hat{i} = 196 \quad \text{for } T_2$$

$$T_1 \hat{i} = -T_2 \hat{i} = 196$$

$$T_1 = -196 \hat{i} + 3.92 \hat{j}$$

$$T_2 = 196 \hat{i} + 3.92 \hat{j}$$

