9/16/09

Outline:

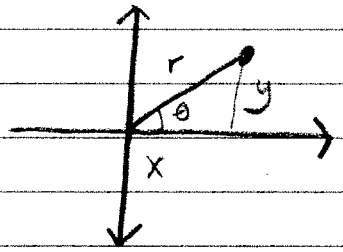
- Polar (Finish 11.3)
- Polar areas (11.4)
  (& arc length)

Announcements:

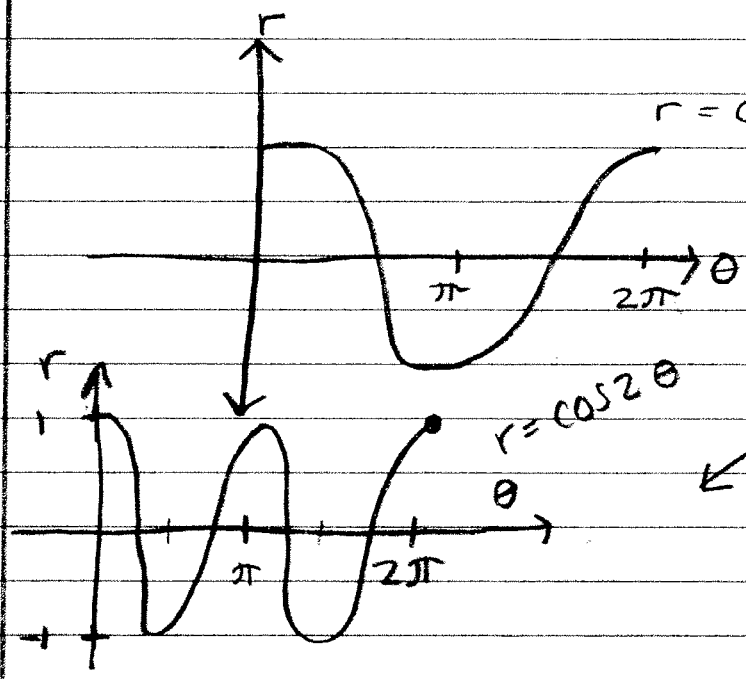
- 9/22 Tuesday, 4PM
Math Grad School Info Discussion
- Shape of Space
  Tuesday Oct. 6, 5-6PM

Sheila M hopes it doesn't rain

Polar Coordinates

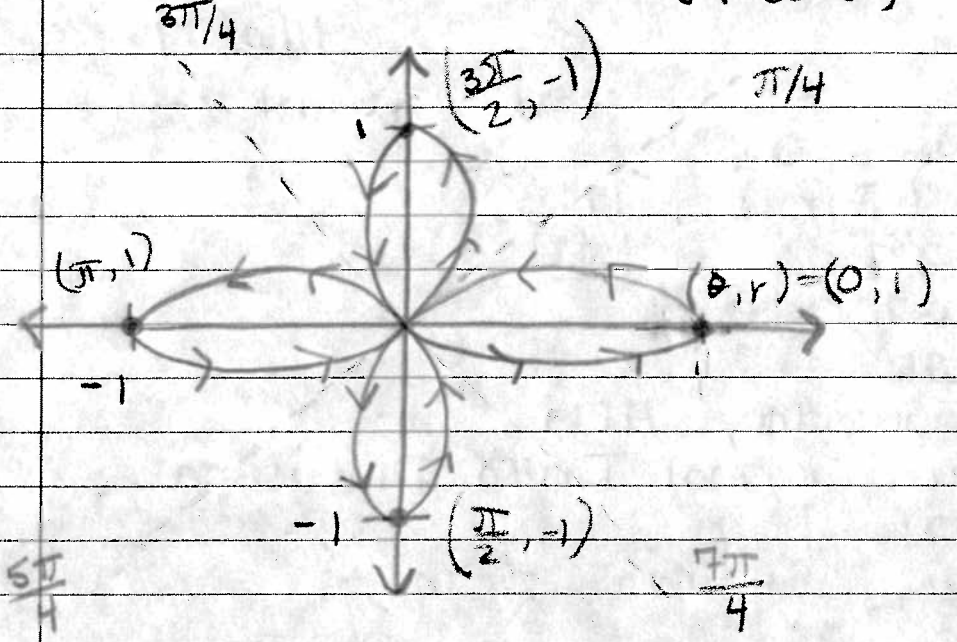


Ex. Sketch $r = \cos 2\theta$



← use this graph to sketch the polar curve.

Roses (4 leaves)



θ	r
0	1
π/4	0
π/2	-1
3π/4	0
π	1
5π/4	0
3π/2	-1
7π/4	0
2π	1

Review (Polar Graphs)

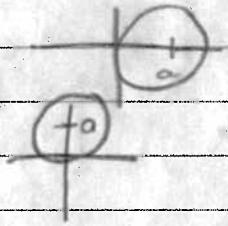
r = constant (circle)

θ = constant (line through origin)

$r = 2a\cos\theta$ →

or

$2a\sin\theta$ →



$r = 1 + \sin\theta$ (cardioid)

$r = \cos n\theta$ (roses)

Calculus on Polar curves

Suppose $r = f(\theta)$

Then

$x(\theta) = r\cos\theta = f(\theta)\cos\theta$

$y(\theta) = r\sin\theta = f(\theta)\sin\theta$

Q: How do we calculate the slope of the tangent to this parametric curve?

A: $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(f(\theta)\sin\theta\right) = \frac{d}{d\theta} f(\theta)\sin\theta + f(\theta)\cos\theta$

$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(f(\theta)\cos\theta\right)$

$= \frac{dr}{d\theta}\cos\theta - r\sin\theta$

So $\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

as long as $\frac{dx}{d\theta} \neq 0$

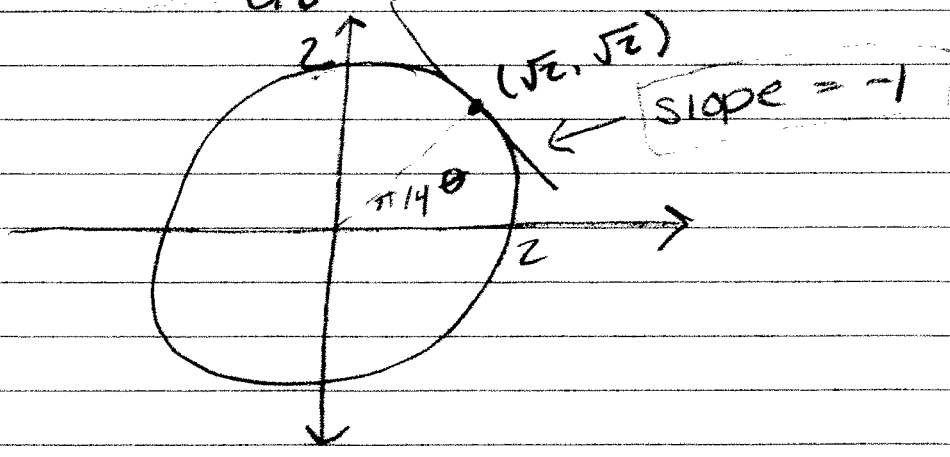
Check for horizontal tangents by checking $\frac{dy}{d\theta} = 0$

and vertical by $\frac{dx}{d\theta} = 0$

Exercise:

$f(\theta) = 2$

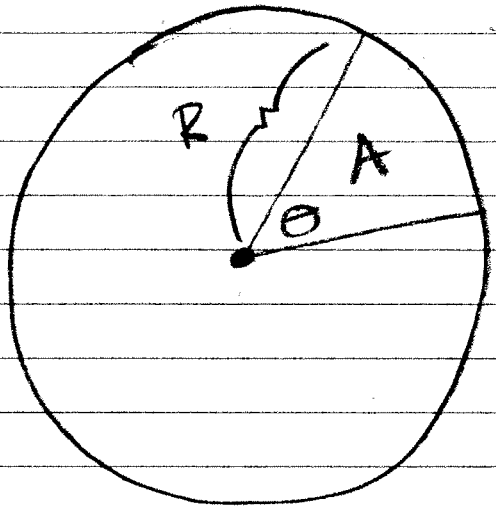
$r = 2$

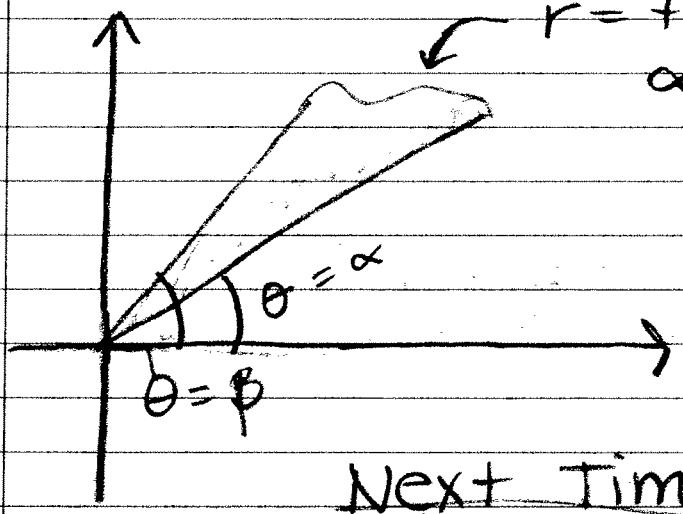


11.4 Polar Areas

$$\text{Area}(A) = \pi R^2 \frac{\theta}{2\pi}$$

$$\frac{R^2\theta}{2}$$

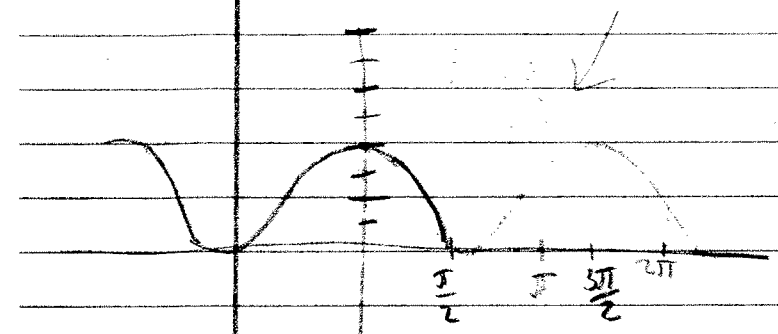


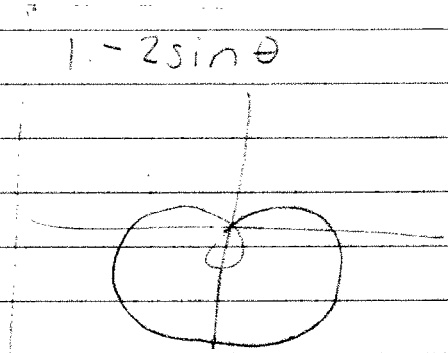


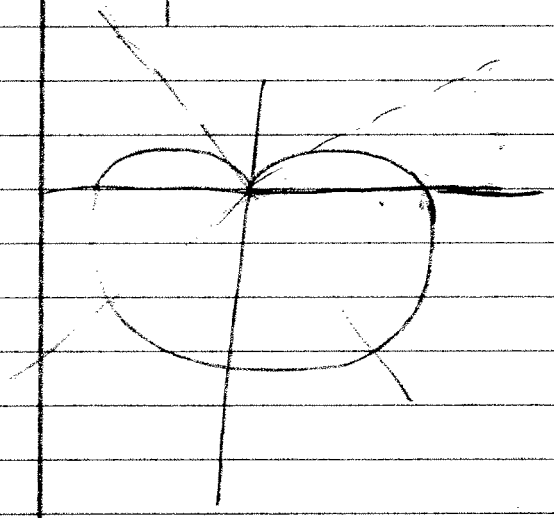
Next Time:
We'll calculate this area

33. $r = 2(1 - \sin\theta)$, $\theta \geq 0$

$2 - 2\sin\theta$, $\theta \geq 0$







θ	r
0	2
π/4	.5858
π/2	0
π	2
5π/4	3.412
3π/2	4
7π/4	3.412
2π	2