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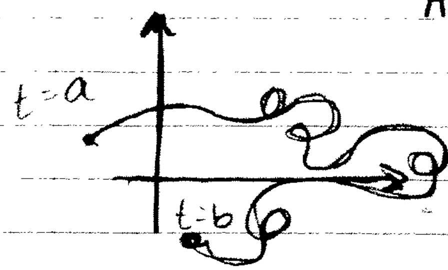
ARC LENGTH



$x = f(t)$,
$y = g(t)$
$(x(t), y(t))$, $a \le t \le b$

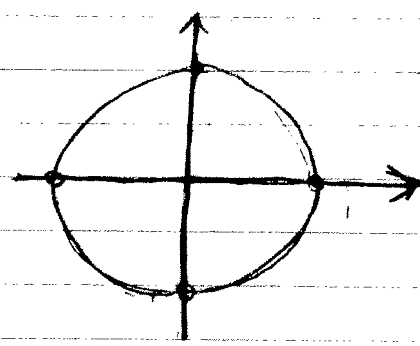
If f' and g' are continuous functions then the length of the curve from $t=a$ to $t=b$ is:

$$\text{Length } L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

provided you only travel once along the curve from $t=a$ to $t=b$

Example

C: $x = \cos t$, $y = \sin t$ $0 \le t \le 2\pi$

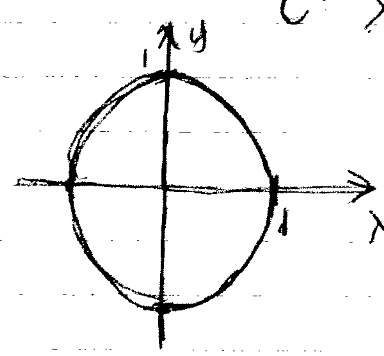


We know that C is 2π long But lets check:

$$l \text{ of } C = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt = \int_0^{2\pi} 1\, dt = t\Big|_0^{2\pi} = 2\pi - 0 = 2\pi$$

$\boxed{\text{EXAMPLE}}$

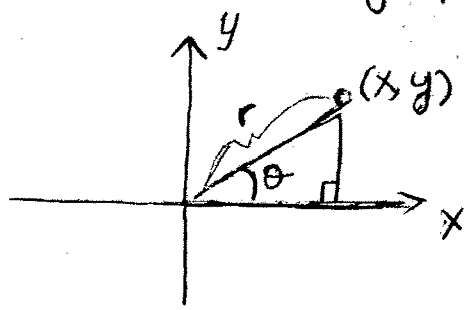
C: $x = \cos 2t$, $y = \sin 2t$



$$= \int_0^{2\pi} \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2}\, dt$$
$$= \int_0^{2\pi} 4\sin^2 2t + 4\cos^2 2t \; dt$$
$$= \int_0^{2\pi} \sqrt{4(\sin^2(2t) + \cos^2(2t))}\, dt$$
$$= \int_0^{2\pi} 2\, dt = 2t\Big|_0^{2\pi} = 2(2\pi) - 0 = 4\pi$$

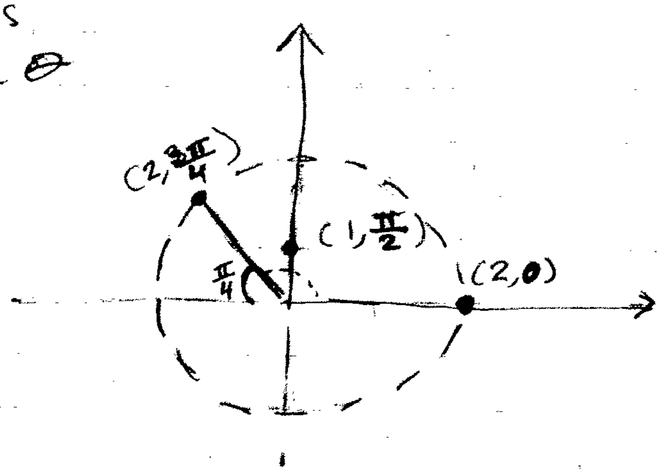
11.3: POLAR COORDINATES

describes the (x,y) plane

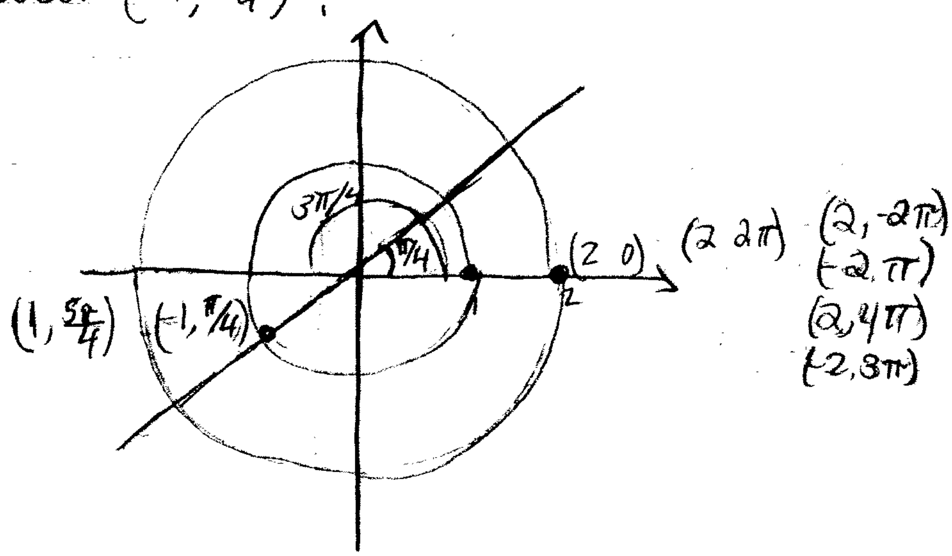


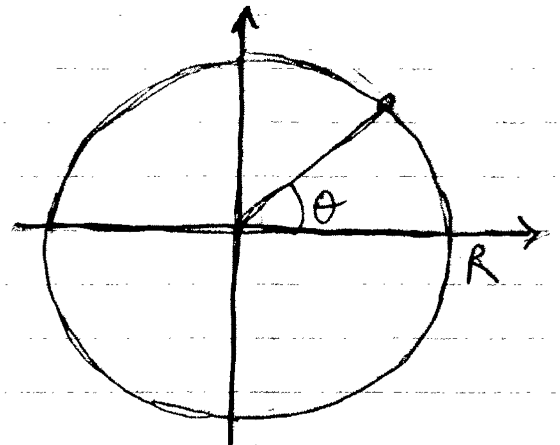
r and θ are Polar Coordinates for the point (x, y)

USE RADIANS to describe θ



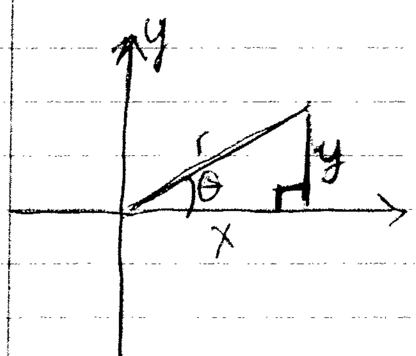
what about $(-1, \frac{\pi}{4})$?





$(R,\theta) = (R, \theta + K2\pi)$ $\quad K \in \mathbb{Z}$ (K can be any integer)
OR
$= (-R, \theta + K\pi)$ $\quad K \in \mathbb{Z}$, that's odd

NOTATION:
(x,y) are called Rectangular or Cartesian coordinates

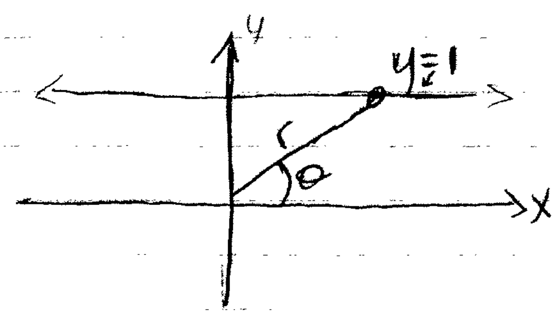


its fundamental to know how x,y,r,θ relate

$x = r\cos\theta \qquad r^2 = x^2 + y^2$
$y = r\sin\theta \qquad \tan\theta = \dfrac{y}{x}$

$\boxed{\text{EXAMPLE}}$

$r\sin\theta = 1$
$r = \frac{1}{\sin\theta}$



This means that $r = f(\theta)$

Compare: $x \longmapsto 1$ (function#1) & $\theta \longrightarrow \frac{1}{\sin\theta}$ (function #2)

The line $y=1$ is the graph of function#1 as $-\infty < x < \infty$
The Line $y=1$ is also the graph of function 2 as $-\infty < x < \infty$

1 SAMPLE PROBLEM : ARC LENGTH

The parametric equations

$x = 6\cos(2t)$

$y = 6\sin(2t)$

describe the circle of radius 6 centered at the origin of the plane

Q: Find the number T for which the length of the parametric curve from $t = 0$ to $t = T$ = 20.

$$L = \int_0^T \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$20 = \int_0^T \sqrt{(-12\sin 2t)^2 + (12\cos 2t)^2}\, dt$$

$$20 = \int_0^T \sqrt{144(\sin^2 2t + \cos^2 2t)}\, dt$$

recall $\sin^2 2t + \cos^2 2t =$

$$20 = 12\int_0^T \sqrt{7}\, dt$$

$$20 = 12(t)\Big|_0^T$$

$$20 = 12(T) - 12(0)$$

$$\frac{20}{12} = T, \quad \boxed{T = \frac{5}{3}}$$