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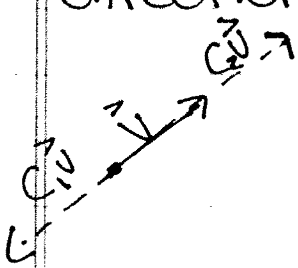
Outline: lines and Planes

Parallel lines

2 lines are $\parallel$ in the plane when they have the same slope
2 lines are ~~pa~~ $\parallel$ in $\mathbb{R}^3$ when they have direction vectors that point the same way



any constant times $\vec{v}$

defn: Two vectors are parallel if they differ from each other by multiplication of a scalar

ex: $\langle 1,1,2\rangle$ & $\langle 2,2,4\rangle$ parallel
$\langle 3,0,-1\rangle$ & $\langle -6,0,2\rangle$ parallel
$\langle 1,2,3\rangle$ & $\langle 0,1,-7\rangle$ not parallel
no c so that $c\cdot 1=0$, $c\cdot 2=1$, & $c\cdot 3=-7$

ex: L_1: $x=1+t$, $y=-2+3t$, $z=4-t$
L_2: $x=2s$, $y=3+s$, $z=-3+4s$
point on $L_1=(1,-2,4)$, direction vector L_1: $\langle 1,3,-1\rangle$
point on $L_2=(0,3,-3)$, direction vector L_2: $\langle 2,1,4\rangle$
Observation - L_1 & L_2 are not $\parallel$, show L_1 & L_2 are skew
Skew - lines that dont intersect and are not $\parallel$

We have to show they dont intersect,
Find a t value and a s value that make the x, y, z coorichates of L_1 and L_2 equal:

① $1+t=2s$
② $-2+3t=3+s$
③ $4-t=-3+4s$

① $s=\frac{1+t}{2}$ plug ① into ②

$-2+3t=3+\left(\frac{1+t}{2}\right)=-4+6t=6+1+t=$

$5t=11$ $\boxed{t=\frac{11}{5}}$

use ① to get s

$s=\frac{1+\frac{11}{5}}{2}=\frac{\frac{5+11}{5}}{2}=\frac{16}{10}=\frac{8}{5}$ $\boxed{s=\frac{8}{5}}$

plug s and t into ③

$4-\frac{11}{5}=-3+4\left(\frac{8}{5}\right)$

$\frac{9}{5}\neq\frac{17}{5}$ $\therefore$ they are skew

ex: Find an equation for the plane through $P(1,3,2)$ $Q(3,-1,6)$ $R(5,2,0)$

~~Aleb~~ solution: A point: $(1,3,2)$

A $\perp$ vector: $\vec{PQ}\times\vec{PR}=\vec{n}$

$\vec{n}=\langle 12,20,14\rangle$

$0=\langle x-1,y-3,z-2\rangle\cdot\langle 12,20,14\rangle$

$0=\underline{12}(x-1)+\underline{20}(y-3)+\underline{14}(z-2)$

came from normal vector, $\vec{n}$

note: $ax+by+cz=d$ always determins a plane with $\vec{n}=\langle a,b,c\rangle$

ex $x+y+z=1$ & $x-2y+3z=1$

Questions:

1. what angle do these planes intersect
2. what is the line of their intersection?

1. The angle of intersection of the planes is the angle between the normal vectors

$\vec{n}_1=\langle 1,1,1\rangle$ $\quad$ $\vec{n}_2=\langle 1,-2,3\rangle$

excercise: find the angle

$a\cdot b=|a||b|\cos\theta$

2. vector: $\vec{n}_1\times\vec{n}_2$

point: set one of x or y or z in the planar equationsequal to 0 and solve

so if $z=0$

$x+y=1 \quad x-2y=1$

$1-y=1+2y$

$0=3y \Rightarrow y=0 \therefore x=1$

So $(1,0,0)$ is contained in both planes so it is contained in their intrsection so it is in the line

answer: find the line w/ $\vec{v}=\vec{n}_1\times\vec{n}_2$ and passing through $(1,0,0)$

p839, #47

Find the direction numbers for the line of intersection of the planes:

$x + y + z = 1 \qquad x + z = 0$

first, we need the two normal vectors

for $x+y+z=1$ we get $\langle 1,1,1 \rangle$

for $x+z=0$ we get $\langle 1,0,1 \rangle$

next we need to take the cross product of these vectors

$$\begin{array}{cccccc} i & j & k & i & j & k \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$1-0,\ 1-1,\ 0-1 = \langle 1,0,-1 \rangle$

$\therefore$ the direction numbers are $1, 0, -1$