Kim Monoogian

Kim M. wants the summer back

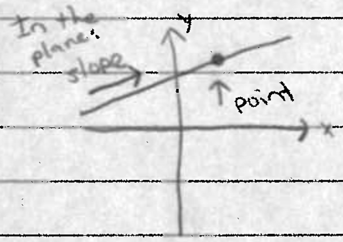
10/7 Outline

Lines / Planes

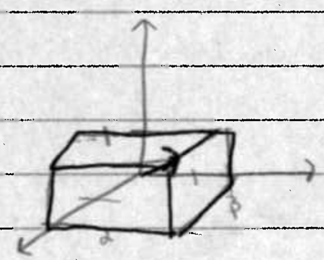
No HW due tomorrow

Quiz on Tuesday on 13.5/13.6

Lines



point and slope determine the line



$\langle 2,3,1\rangle$

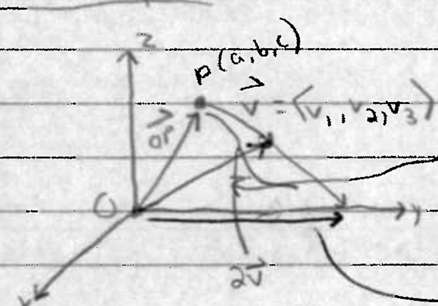
In 3-D space, the data you need is: a point $P\ (a,b,c)$

a vector $\vec{v} = \langle v_1, v_2, v_3\rangle$

① Parametric description of a line:

② Vector Description

Vector Description



$\vec{OP} = \langle a,b,c\rangle$

$\vec{OP} + \vec{v}$

$\vec{OP} + 1\cdot\vec{v}$

$\vec{OP} + 2\cdot\vec{v}$

The vector description of the line through $P(a,b,c)$ and in the direction $\vec{v}$ is $\vec{r}(t) = \vec{OP} + t\,\vec{v} \quad t \in \mathbb{R}$

$\langle a,b,c\rangle + t\langle v_1, v_2, v_3\rangle = \langle a+tv_1, b+tv_2, c+tv_3\rangle$

Parametric description

Parametric equations of the line are:

$x(t) = a + tv_1$

$y(t) = b + tv_2$

$z(t) = c + tv_3$

Example: $P(2,5,-3)$

$\vec{v} = \langle 7, 12, 9 \rangle$

line:

$\vec{r}(t) = \langle a+tv_1, b+tv_2, c+tv_3 \rangle$

$= \langle 2+7t, 5+12t, -3+9t \rangle$ - vector form

parametric form

$x = 2+7t$ t can be any real number

$y = 5+12t$ or

$z = -3+9t$ $t \in \mathbb{R}$

Symmetric equations of a line:

Obtain these by solving all the parametric equations for t:

$\frac{x-2}{7} = t$

$\frac{y-5}{12} = t$

$\frac{z+3}{9} = t$

Symmetric equations:

$$\frac{x-2}{7} = \frac{y-5}{12} = \frac{z+3}{9}$$

Example:

$\frac{x+1}{7} = \frac{y-3}{-2} = z$ Write down the parametric equations for this line

Sol'n: $\frac{x+1}{7} = t$

$\frac{y-3}{-2} = t$

$z = t$

$\xrightarrow{\text{solve for } x,y,z}$

$x = -1 + 7t$

$y = 3 - 2t$

$z = 0 + 1t$

point on line

components of vector

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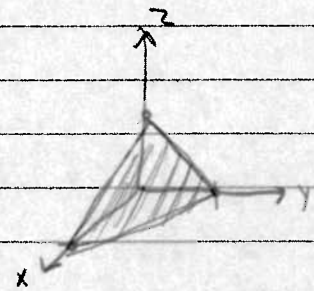
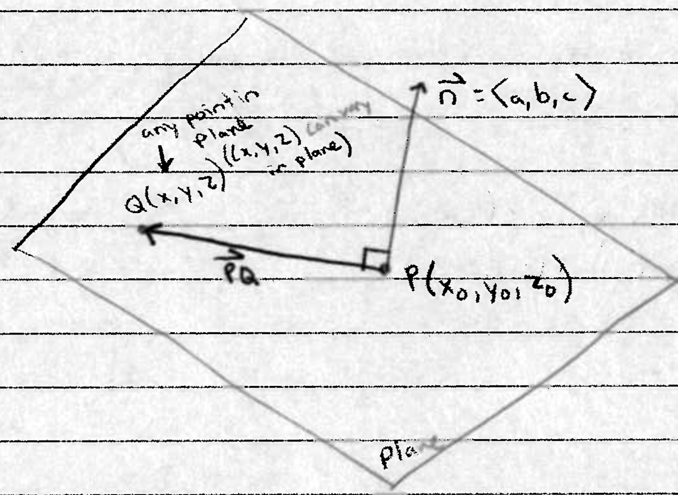
Planes



Data

- Point in the plane $P(x_0, y_0, z_0)$
- vector $\vec{n} = \langle a, b, c \rangle$ $\perp$ to the plane



$\boxed{\vec{PQ} \cdot \vec{n} = 0}$ b/c they are $\perp$

$$\vec{PQ} = \langle x - x_0, y - y_0, z - z_0 \rangle$$
$$\vec{n} = \langle a, b, c \rangle$$
$$\vec{PQ} \cdot \vec{n} = \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

equation of a plane passing through (x_0, y_0, z_0) and $\perp$ to $\langle a, b, c \rangle$.

Example

$P = (1, 2, 3)$

$\vec{n} = \langle 1, 1, -2 \rangle$

The plane has equation:

$$\boxed{1(x-1) + 1(y-2) - 2(z-3) = 0}$$

This means that a point (x, y, z) is in this plane if and only if x, y, z satisfy this equation.

$x = a$

$1(x-a) + 0(y-0) + 0(z-0) = 0$

equation of plane passing through the point $(a,0,0)$ and vector $\langle 1,0,0\rangle$

$\boxed{\text{Sample Problem}}$

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Find an equation of the plane through the point $(1,-1,1)$ and with normal vector $i+j-k$.

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$ $\qquad \vec{n} = \langle 1,1,-1\rangle = \langle a,b,c\rangle$

$1(x-1) + 1(y-(-1)) + (-1)(z-1) = 0$ $\qquad$ point $= (1,-1,1) = (x_0,y_0,z_0)$

$x - 1 + y + 1 - z + 1 = 0$

$\boxed{x + y - z = -1}$