Stacey Leonardo Sanchez   10/22/09

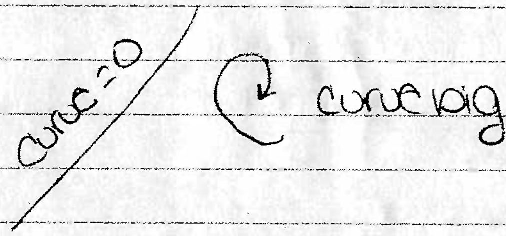
Stacey L. is excited for a possible yankees world series!

Outline
- Curvature (for space curves)

Announcements
- Exam II Thursday October 29
- Wed probably for review
- T. Oct 27 5pm Library Multimedia room "scary math"

Curvature: (This is a measure of how much a curve is turning at a given point)

We'll see:



Concept
Different parametrization of a curve.

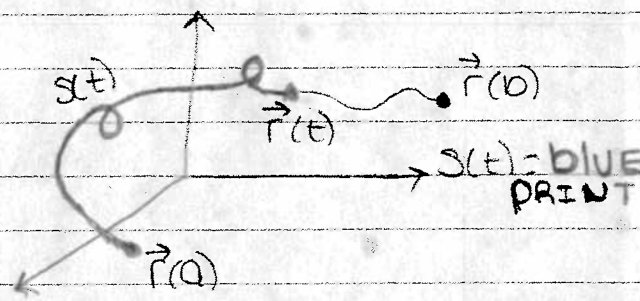
Example: $\vec{r}(t) = \langle t, t^2, t^3 \rangle \quad 1 \le t \le 2$
one space curve

$$\vec{r}(u) = \langle e^u, e^{2u}, e^{3u} \rangle \quad 0 \le u \le \ln 2$$

- these are both curves from (1,1,1) to (2,4,8)
- In fact, the space curves as geometric objects are identical. The only way they differ is by the speed at which they are traversed

Define: the arc length function for a curve $\vec{r}(t)$ $(a \le t \le b)$

$$s(t) = \int_a^t \underbrace{|\vec{r}\,'(u)|}_{\text{speed}}\,du$$

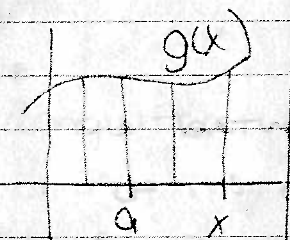


① $\frac{ds}{dt} = |\vec{r}\,'(t)|$ (The differentiation of s(t) using the fundamental theorem of calculus)

Recall calc 2:

$$f(x) = \int_a^x g(u)\,du \xrightarrow[\text{FTC}]{} f'(x) = g(x)$$

(fundamental theorem of calculus)



② very often the function

$$s(t) = \int_a^t |\vec{r}\,'(u)|\,du$$

can be solved for t.

$t(s) = t$

then $\vec{r}(t(s))$ is called parametrized by arc length.

param'd by arc length - in s units of time, you travel a distance of s units of length.

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle \quad 0 \le t$

$$s(t) = \int_0^t |\vec{r}\,'(u)|\,du$$

$$= \int_0^t \sqrt{(-\sin u)^2 + (\cos u)^2 + (1)^2}\;du$$

$$= \int_0^t \sqrt{2}\;du$$

$$= \sqrt{2}u \Big|_0^t = \sqrt{2}\,t$$

$s(t) = \sqrt{2}\,t \rightarrow t = \dfrac{s}{\sqrt{2}}$ so $\boxed{t(s) = \dfrac{s}{\sqrt{2}}}$

$\boxed{\vec{r}(t(s)) = \left\langle \cos\left(\dfrac{s}{\sqrt{2}}\right), \sin\left(\dfrac{s}{\sqrt{2}}\right), \dfrac{s}{\sqrt{2}} \right\rangle}$ A new parametrizatio

Lets calculate the arc length of $\vec{r}(s)$ from $s=0$ to $s=2\pi$:

$$\int_0^{2\pi} |\vec{r}\,'(s)|\,ds = \int_0^{2\pi} \sqrt{\left[\frac{1}{\sqrt{2}} \cdot -\sin\left(\frac{s}{\sqrt{2}}\right)\right]^2 + \left[\frac{1}{\sqrt{2}} \cdot \cos\left(\frac{s}{\sqrt{2}}\right)\right]^2 + \left(\frac{1}{\sqrt{2}}\right)^2}\;ds$$

$$\int_0^{2\pi} \sqrt{\frac{1}{2}\sin^2\left(\frac{s}{\sqrt{2}}\right) + \frac{1}{2}\cos^2\left(\frac{s}{\sqrt{2}}\right) + \frac{1}{2}}\;ds$$

$$\int_0^{2\pi} \sqrt{\frac{1}{2} + \frac{1}{2}}\;ds = \int_0^{2\pi} 1\,ds = 2\pi - 0 = \boxed{2\pi}$$

$\vec{r}(s)$ is parametrized so that you travel s units of distance in s units of time

↓

b/c $|\vec{r}\,'(s)| = 1$

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(13) $r(t) = 2t\,i + (1-3t)\,j + (5+4t)\,k$
Reparametrize this curve with respect to arc length measured from the point where $t=0$ in the direction of increasing t

Arc length function: $s(t) = \int_a^t |r'(u)|\,du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2}\,du$

we will use this formula because $s(t)$ gives us a new t, which forms the new parametrization we are looking for.

$r(t) = \langle 2t,\ 1-3t,\ 5+4t \rangle$

$\frac{dx}{du} = 2 \quad \frac{dy}{du} = -3 \quad \frac{dz}{du} = 4$

plug these values into the arc length function

$$s(t) = \int_0^t \sqrt{(2)^2 + (-3)^2 + (4)^2}\,du$$

a becomes zero because we are looking for the length of the arc traced after $t=0$ as t increases

$$s(t) = \int_0^t \sqrt{4+9+16}\,du$$

$$s(t) = \int_0^t \sqrt{29}\,du$$

$s(t) = \sqrt{29}\, u \Big|_0^t \rightarrow s(t) = \left[\sqrt{29}(t) - \sqrt{29}(0)\right]$

$s(t) = \sqrt{29}\, t$ now that we have the arclength function as a function of t, we will solve for t as a function s

$\frac{s}{\sqrt{29}} = \frac{\cancel{\sqrt{29}}\, t}{\cancel{\sqrt{29}}} \rightarrow t = \frac{s}{\sqrt{29}}$ This is t as a function of s, the arclength function

essentially,

$t(s) = \frac{s}{\sqrt{29}}$ and this will be the new t for the new vector function (our parametrized function)

plug this t using this formula: $\vec{r}(t(s))$, meaning whenever you see a t in the original vector, plug in $t(s)$

$\vec{r}(t) = 2t\,i + (1-3t)\,j + (5+4t)\,k$

$\vec{r}(t) = \langle 2t, 1-3t, 5+4t \rangle$

$\vec{r}(t(s)) = \left\langle 2\left(\frac{s}{\sqrt{29}}\right), 1-3\left(\frac{s}{\sqrt{29}}\right), 5+4\left(\frac{s}{\sqrt{29}}\right) \right\rangle$

$\vec{r}(t(s)) = \left\langle \frac{2}{\sqrt{29}} s, 1 - \frac{3}{\sqrt{29}} s, 5 + \frac{4}{\sqrt{29}} s \right\rangle$

$$\boxed{r(t(s)) = \left(\frac{2}{\sqrt{29}} s\right) i + \left(1 - \frac{3}{\sqrt{29}} s\right) j + \left(5 + \frac{4}{\sqrt{29}} s\right) k}$$

The reparametrization of the cure