Vector Functions
October 15, 2009

Announcements:
- NO HW due next week! :)
- No quiz on Monday
- Quiz THURSDAY.

Kim hates the rain but is SUPER excited to go hom for the weekend!

§ 14.1 Vector Functions

$t \longmapsto$ vector
also think of these as space curves.

$$\vec{r}(t) = \underbrace{\langle f(t), g(t), h(t) \rangle}_{\substack{\text{end point of } \vec{r}(t) \\ \text{describes a posistion} \\ \text{in space.}}}$$

• Calculus on vector functions:

$$\lim_{t \to a} \vec{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

- space curve

tracing out a path in space.

Ex: $\vec{r}(t) = \langle 1 + t^3, te^{-t}, \frac{\sin t}{t} \rangle$

Q: $\lim_{t \to 0} \vec{r}(t) = \langle 1+0, 0 \cdot 1, 1 \rangle = \langle 1, 0, 1 \rangle$

* Geometrically, this says near time $t=0$, the curve determined by $\vec{r}$ is close to the pt. $(1,0,1)$.

$\boxed{EX:}$ Find a vector function that represents the line segment from $P(1,0,2)$ to $Q(3,-1,0)$

- What do I need to define a line?

Point + Vector

$P(1,0,2)$

$\vec{v} = \vec{PQ} = \langle 3-1, -1, -2\rangle = \langle 2, -1, -2\rangle$

Our answer should look like this:

$\vec{r}(t) = \langle 1+2t, -t, 2-2t\rangle$

$0 \leq t \leq 1$

$\vec{r}(0) = P$

$\vec{r}(1) = Q$

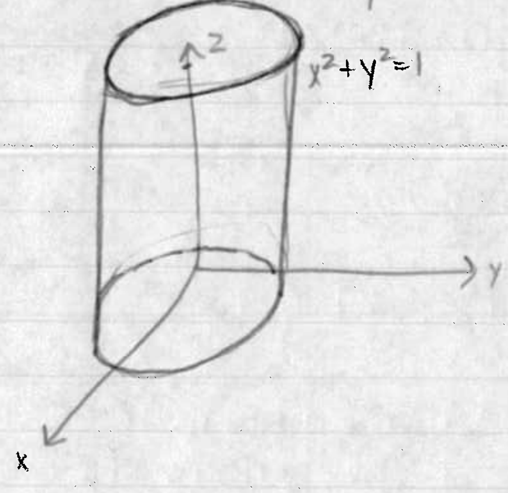
from P to Q,

if we wanted to go from Q to P,

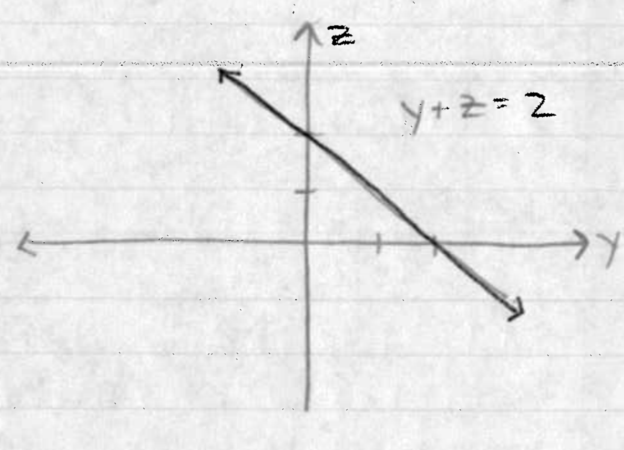
$\vec{r}(t) = \langle 3-2t, -1+t, 2t\rangle$

$0 \leq t \leq 1$

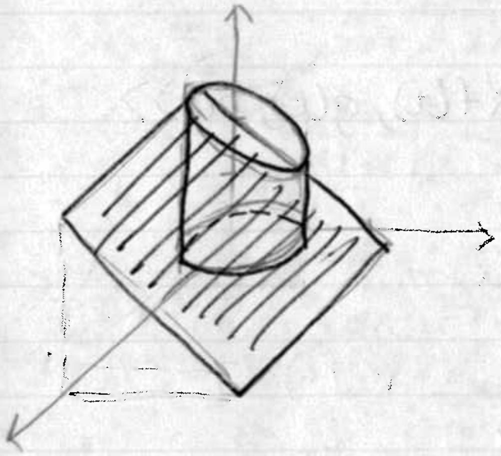
$\boxed{EX:}$ Find a vector function that represents the intersection of the plane $y+z=2$ and the cylindar $x^2+y^2=1$



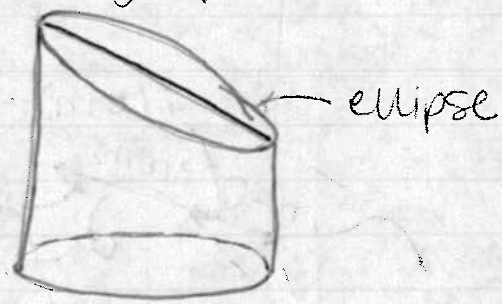
$x=0$ trace



plane in 3D:



Shape of graph:



Using $x^2+y^2=1$ try $\left.\begin{array}{l} x=\cos t \\ y=\sin t \end{array}\right\}$ this will ensure the curve lies on the cylinder

Using this and the fact $y+z=2$, we can write down a function for z.

$$z=2-\sin t$$

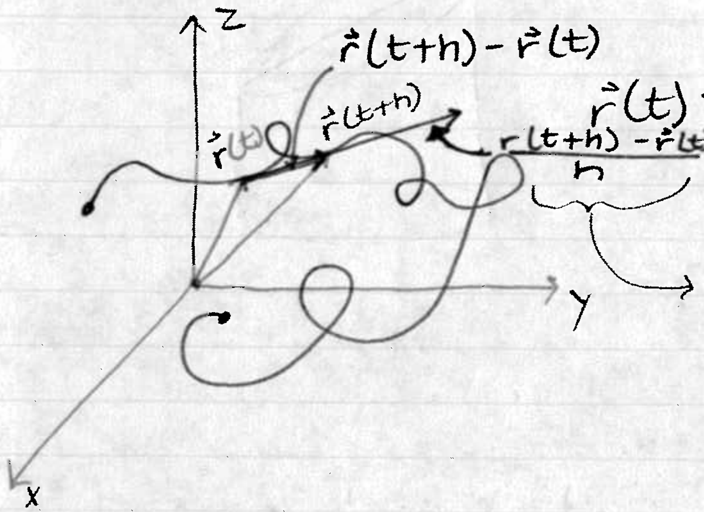
$$\vec{r}(t)=\langle \cos t,\ \sin t,\ 2-\sin t\rangle$$

$$0\leq t\leq 2\pi$$

HW Problems to think about: (not to hand in).

§14.1 # 1, 2, 4, 5, 7, 9, 10, 16, 19-24, 25, 26, 42.

§ 14.2



$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$a \leq t \leq b$

$\lim \dfrac{\vec{r}(t+h)-\vec{r}(t)}{h}$

$= \vec{r}'(t)$ or $\vec{v}(t)$ or $\frac{d\vec{r}}{dt}$

* This is the derivative vector function of $\vec{r}(t)$

$\vec{r}'(t)$ is the tangent vector to the spacecu

Fact:

$\vec{r}(t) = \langle f'(t), g'(t), h'(t) \rangle$

$\vec{T}(t) = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} \longrightarrow$ unit tangent vector function.

$\boxed{EX:}$ $(1+t^3)\vec{i} + (te^{-t})\vec{j} + (\sin(2t))\vec{k}, \quad t \in \mathbb{R}.$

$\vec{r}'(t) = 3t^2\vec{i} + [-te^{-t} + e^{-t}]\vec{j} + 2\cos(2t)\vec{k}$

$= \langle 3t^2, e^{-t} - te^{-t}, 2\cos(2t) \rangle$

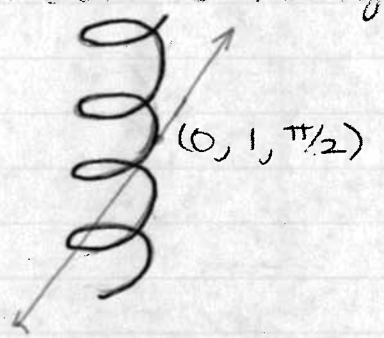
$\vec{T}(t) = \dfrac{1}{\sqrt{(3t^2)^2 + (e^{-t}-te^{-t})^2 + (2\cos(2t))^2}} \, r'(t)$

$T(0) = \dfrac{r'(0)}{|r'(0)|} = \dfrac{\langle 0,1,2 \rangle}{|\langle 0,1,2 \rangle|} = \langle 0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

$\boxed{\text{EX:}}$ Application

$r(t) = \langle 2\cos t, \sin t, t \rangle$

Find the parametric equations for the line which is tangent to this curve at $(0, 1, \pi/2)$



What do I need to find this line?

Point $\rightarrow$ $(0, 1, \pi/2)$ given

Vector $\rightarrow$

Exercise: (finish)

- Compute $r'(t)$.

  $t = \pi/2$ at $(0, 1, \pi/2)$, so plug in $\pi/2$ and use the result of the vector.

- also, read the big box of properties of the derivatives.