Dorothy Montick — 10/14/09

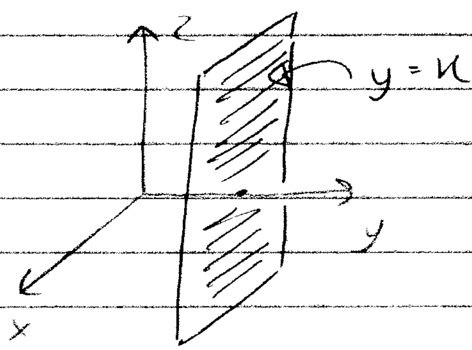
$z = 4x^2 + y^2$ (elliptic paraboloid)

- last time, the horizontal traces $z = k$ look like ellipses
- Now, $y = k$ (vertical traces)

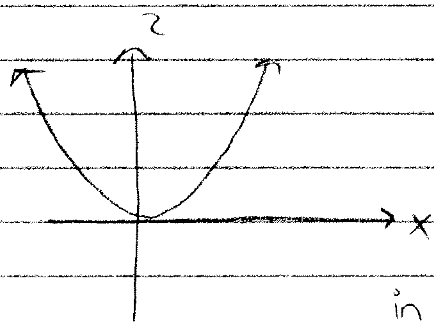
Outline

- Finish Quadric Surfac
- Vector Functions (14)

HW Thurs. Oct 7/8

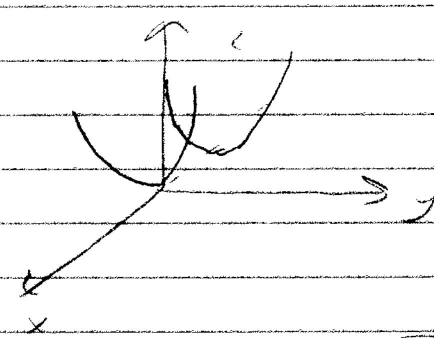


$y = 0$:

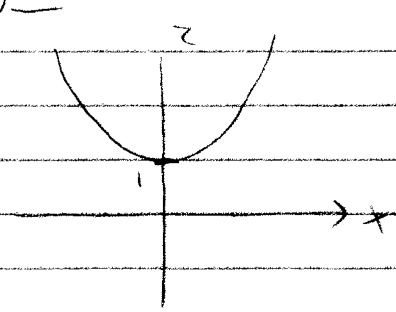


$z = 4x^2$

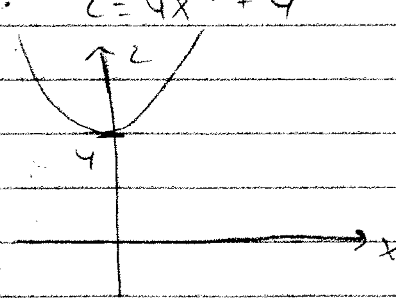
in $\mathbb{R}^3$:

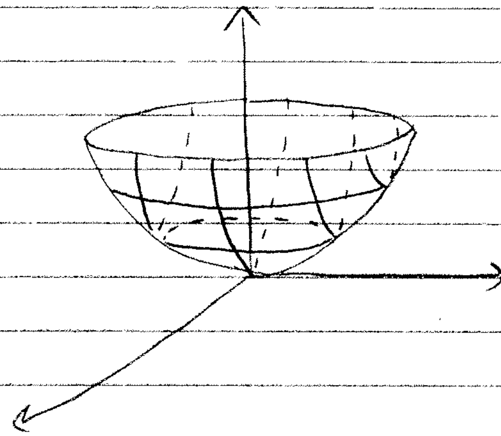


$y = 1$: $z = 4x^2 + 1$



$y = 2$: $z = 4x^2 + 4$





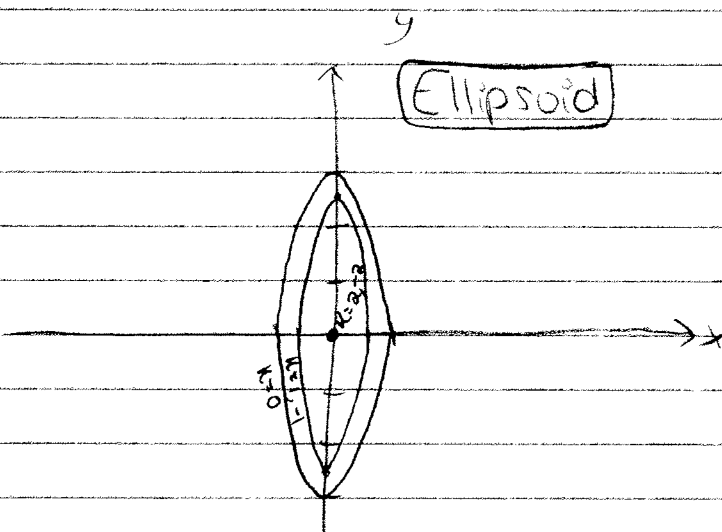
$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

horizontal traces $z = k$

k	$x^2 + y^2/9 + k^2/4 = 1$
0	$x^2 + y^2/9 = 1$
± 1	$x^2 + y^2/9 + 1/4 = 1$
± 2	$x^2 + y^2/9 + 1 = 1$
± 3	$x^2 + y^2/9 + 9/4 = 1$

the traces at height $z = k$ are the same at height $z = -k$.

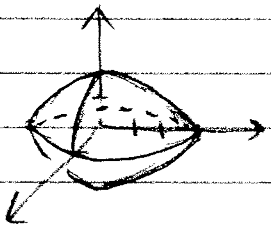
Ellipsoid



$k = \pm 1$:
$x^2 + y^2/9 = 3/4$
$y = 0 \quad x = \pm\sqrt{3}/2$
$x = 0 \quad y = \pm\sqrt{27}/2$

$k = \pm 2$:
$x^2 + y^2/9 = 0$

$k = \pm 3$:
$x^2 + y^2/9 = -5/4$



contained between heights -2 and 2.

exercise:

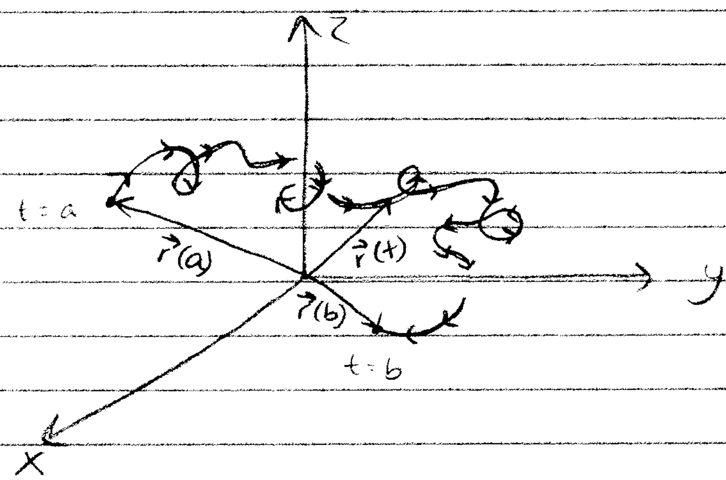
What is the shape of $z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$?

start with $a = 1 = b$

paraboloids, ellipsoids and this object!

HW: 13.6/ 1-7, 11-14, 41, 42

14.1: Vector Functions



Vector Functions describe paths (or curves) in 3-space.

$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$a \le t \le b$

determined by the endpoint of the vector $\vec{r}(t)$.

In parametric form:

$x = f(t)$

$y = g(t) \quad a \le t \le b$

$z = h(t)$

$\vec{r}(t) = \langle 1 + 2t, -1 + 3t, -t \rangle$ ← line through $(1, -1, 0)$ with direction $\langle 2, 3, -1 \rangle$

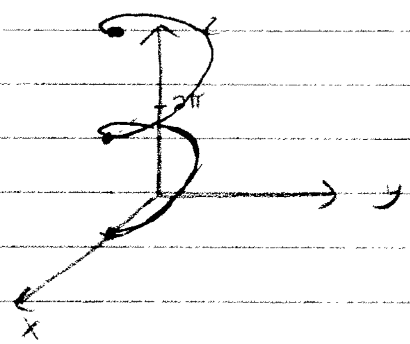
$t \in \mathbb{R}$

$x = 1 + 2t$

$y = -1 + 3t$

$z = 0 - t$

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle \quad 0 \le t \le 4\pi$



spiral! we observe this by analyzing what its xy-motion looks like (circular) and then apply what its z motion looks like.

Example Problem:

14.1/35

35 Show that the curve with parametric equations
$x = t^2$ passes through the points $(1, 4, 0)$ and
$y = 1 - 3t$ $(9, -8, 28)$ but not $(4, 7, -6)$
$z = 1 + t^3$

$\vec{r}(t) = \langle t^2, 1-3t, 1+t^3 \rangle$

$(1, 4, 0)$: $(-1)^2 = 1$ $\quad 1-3t = 4$, $-3t = 3$, $t = -1$ $\quad 1+(-1)^3 = 0$

$(9, -8, 28)$: t^2: $3^2 = 9$ $\quad 1-3t = -8$, $-3t = -9$, $t = 3$ $\quad 1+27 = 28$

but not $(4, 7, -6)$

$(-2)^2 = 4$ $\quad 1-3t = 7$, $-3t = 6$, $t = -2$ $\quad 1 + -8 = -7$ $\quad 1 + -8$ is not -6.