Scribe Notes

11/4/09

Outline

* Motion in Space
* Functions of more than 1 variable
* Turn in 14.3 HW

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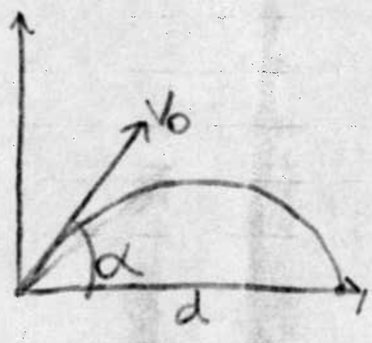

$\cancel{\vec{a}(t) = -mg\vec{j}}$

go back in notes and alter

to: $\vec{a}(t) = -g\vec{j}$

a. $\vec{r}(t) = \langle (v_0 \cos\alpha)t, (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \rangle$

b. Found "d" by solving $(v_0 \sin\alpha)t - \frac{1}{2}gt^2 = 0$

c. $t = \frac{2v_0 \sin\alpha}{g}$ and $t = 0$

d. Plugged t into the x-coordinate (of $\vec{r}(t)$)

e. $d = (v_0 \cos\alpha)\frac{2v_0 \sin\alpha}{g} = \frac{2v_0^2 \cos\alpha \sin\alpha}{g}$

f. Take the derivative of d w/respect to α to maximize d!

g. $\frac{2v_0^2}{g}\left[-\sin^2\alpha + \cos^2\alpha\right] = 0$ and only α that satisfies $\sin^2\alpha = \cos^2\alpha$ is $\alpha = \pi/4$

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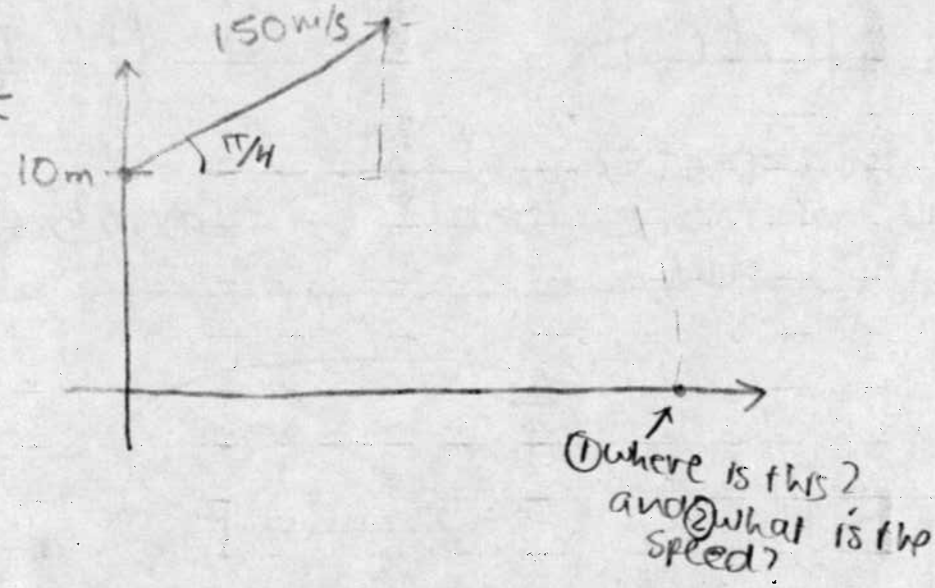


a. $\vec{r}_0 = \vec{r}(0) = 10\vec{j} = \langle 0, 10 \rangle$

b. $\vec{v}_0 = 150\cos(\frac{\pi}{4})\vec{i} + 150\sin(\frac{\pi}{4})\vec{j} =$
$\langle 75\sqrt{2}, 75\sqrt{2} \rangle$

c. $|\vec{v}_0| = v_0 = 150$

d. $\vec{a}(t) = -g\vec{j}$

e. $\vec{v}(t) = \int \vec{a}(t)\, dt = -gt\vec{j} + \vec{c}$

f. $\langle 75\sqrt{2}, 75\sqrt{2} \rangle = \vec{v}_0 = \vec{c}$

g. $\therefore \vec{v}(t) = \langle 75\sqrt{2}, 75\sqrt{2} - gt \rangle$

h. $\vec{r}(t) = \int \vec{v}(t)\, dt = \langle 75\sqrt{2}\, t, 75\sqrt{2}\, t - \frac{gt^2}{2}$
$+ \vec{c}$

i. $\langle 0, 10 \rangle = \vec{r}_0 = \vec{c}$

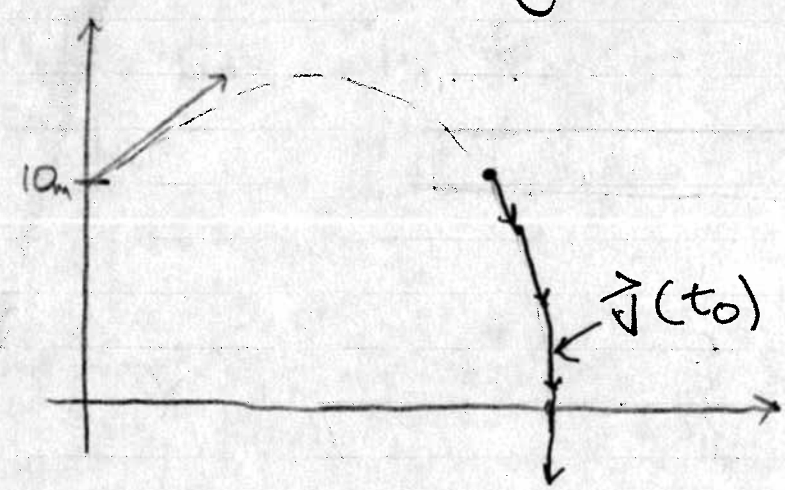
j. $\vec{r}(t) = \langle 75\sqrt{2}\, t, 75\sqrt{2}\, t - \frac{gt^2}{2} + 10 \rangle$

a. ① Where is the impact?

b. Find t where $75\sqrt{2}\,t - \frac{gt^2}{2} + 10 = 0$ so
is ≈ 21.74 and plug into $75\sqrt{2}\,t_0$

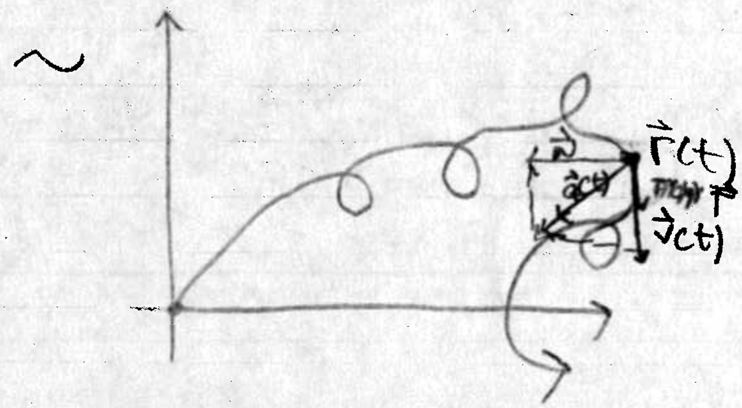
a. ② What is the speed of impact?

b. $\vec{v}(t_0)$ = velocity vector @ impact time



speed $= |\vec{v}(t_0)| \approx 151$ m

* Acceleration



~ It is ~~often~~ useful to resolve
$\vec{a}(t)$ into $(?)\vec{T} + (?)\vec{N}$ so find ?'s

* $\vec{a}(t) = \vec{v}'(t)$

a. $\vec{T} = \frac{\vec{v}}{|\vec{v}|} \rightsquigarrow \vec{v} = |\vec{v}|\vec{T} = v\vec{T}$

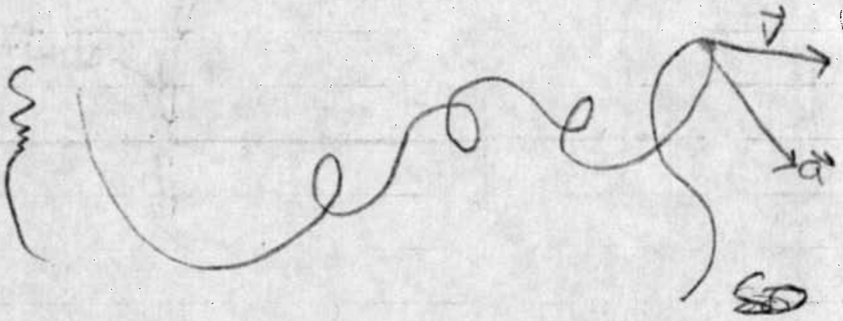
b. $\vec{a} = \vec{v}' = (v\vec{T})' = v'\vec{T} + v\vec{T}'$

c. of course $\frac{\vec{T}'}{|\vec{T}'|} = \vec{N} \longrightarrow T' = |\vec{T}'|\vec{N}$

d. so $\vec{a} = v'\vec{T} + v|\vec{T}'|\vec{N}$ and $K = \frac{|\vec{T}'|}{v}$

e. so $\boxed{\vec{a} = v'\vec{T} + v^2K\vec{N}}$

this means



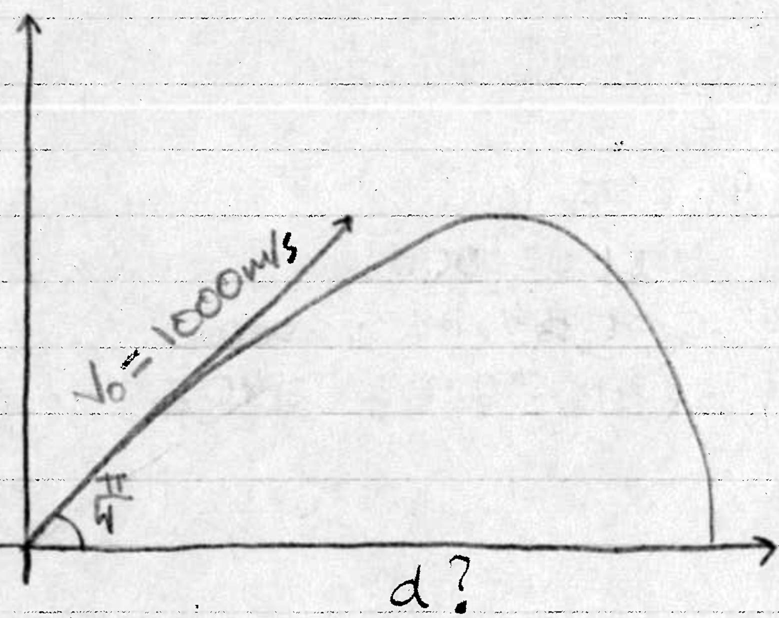
so

If we assume we are on a straight line: $\vec{a} = v'\vec{T}$ b/c $K = 0$, but if we are on a sharp curve then $\vec{a} = v'\vec{T} + v^2K\vec{N}$ which says traveling on a sharp curve you feel what you do b/c of the $\vec{a}$ formula.

Scribe Problem:

Q: A cannon is fired with an initial speed of 1000 m/s at an initial angle of 45 degrees from the ground. Find the initial position vector and the point of impact

A:



Steps for initial position vector

① $\vec{r}(0) = \langle 0, 0 \rangle$

② $\vec{v}(0) = \langle 1000\cos(\frac{\pi}{4}), 1000\sin(\frac{\pi}{4}) \rangle =$
$\langle 500\sqrt{2}, 500\sqrt{2} \rangle$

③ $\vec{v}(t) = -9.8t\vec{j} + \vec{C}$

④ $\vec{C} = \langle 500\sqrt{2}, 500\sqrt{2} \rangle$

(5) $\vec{v}(t) = \langle 500\sqrt{2},\ 500\sqrt{2} - 9.8t \rangle$

(6) $\vec{r}(t) = \langle 500\sqrt{2}\,t,\ 500\sqrt{2}\,t - \frac{9.8t^2}{2} \rangle + \vec{C}$

(7) $\vec{r}(0) = \langle 0, 0 \rangle = \vec{C}$

(8) $\vec{r}(t) = \langle 500\sqrt{2}\,t,\ 500\sqrt{2}\,t - \frac{9.8t^2}{2} \rangle$

Steps for Point of Impact

(1) $500\sqrt{2}\,t - \frac{9.8\,t^2}{2} = 0$

(2) $-\frac{9.8\,t^2}{2} = -500\sqrt{2}\,t$

(3) $\frac{9.8t^2}{2} = 500\sqrt{2}\,t$

(4) $9.8t^2 = 1000\sqrt{2}\,t$

$9.8t = 1000\sqrt{2}$

(5) $t_0 \approx 144.31$ seconds

(6) $500\sqrt{2}(144.31) \approx 102402.58$ meters