9:30 - 10:45 AM
Thurs Sept. 3rd

Differential Calculus ♡

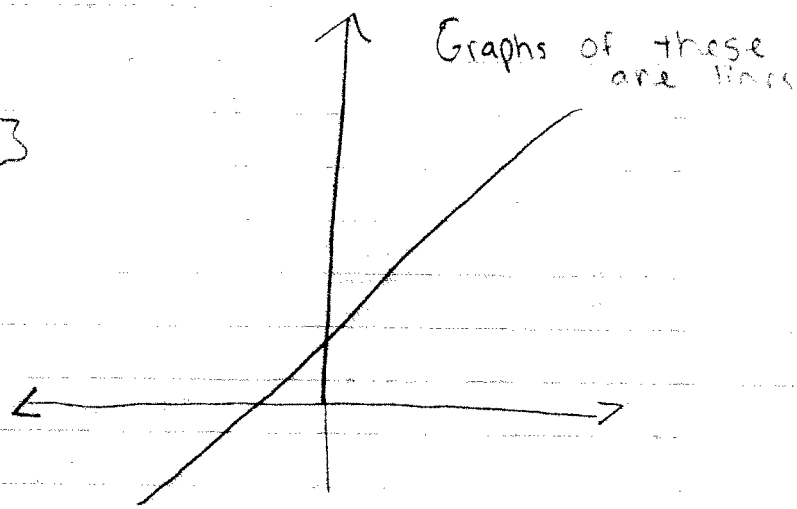
* Announcements

- Make sure check website often
faculty.fairfield.edu/SRafalski
- Make sure dress nice next wk
for "Math Department Photographs"
- Mark Tues Oct. 6th 5-6
"Shape of Space" lecture
part of grade

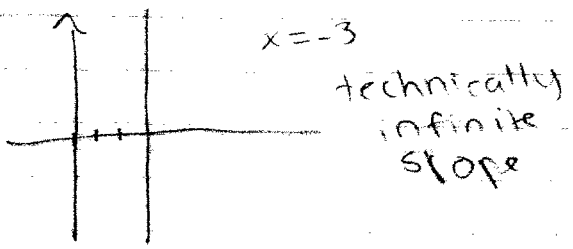
Outline
FUNCTIONS!

Ex. Fnxs

① Linear fnx
$f(x) = \{\#3\} \cdot x + \{\#2\}$
e.g. $2x + 1$
$-3.5x + 720$
$\pi x + \sqrt{\pi}$
$0x + 1 = 1$



* Every line is graph of a linear fnx
EXCEPTION = vertical lines
↳ slope undefined bc it is infinite

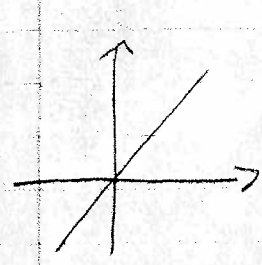


1) Power Fnx

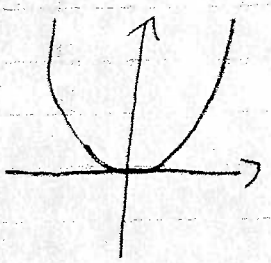
$g(x) = x^k$

if x is positive integer

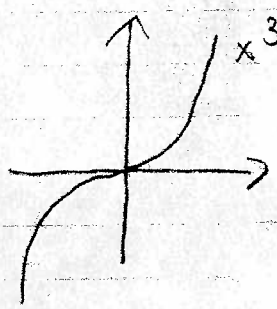
$x, x^2, x^3, x^4, \ldots$



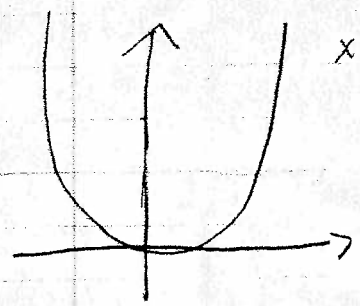
$x = g(x)$



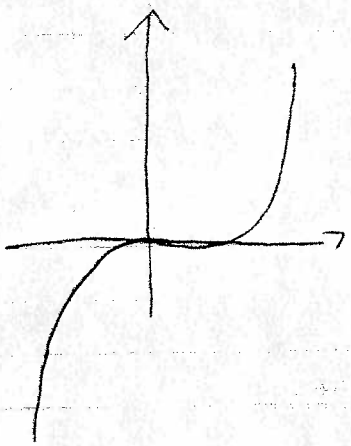
x^2



x^3



x^4



x^5

Proper Notation

$y = f(x)$

- y: ind. variable, "output"
- f: rule
- x: input

We usually have algebraic rule for the function:

for example:

$f(x) = x^3 + 2x + 1$

$f(3) = 3^3 + 2(3) + 1 = 34$

$f(a+h) = (a+h)^3 + 2(a+h) + 1$

NVR $= f(a+h) = f(a) + f(h)$

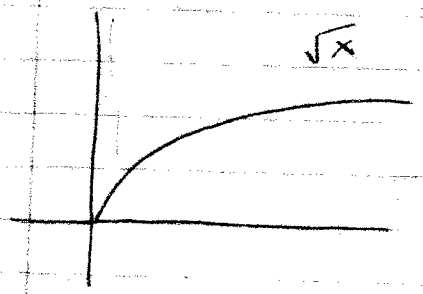
More Power Fnxs

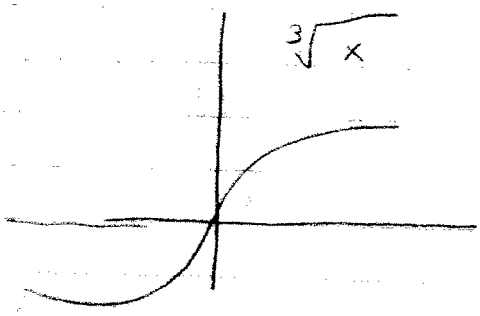
$f(x) = x^k \qquad k = \frac{1}{\text{positive integer}}$

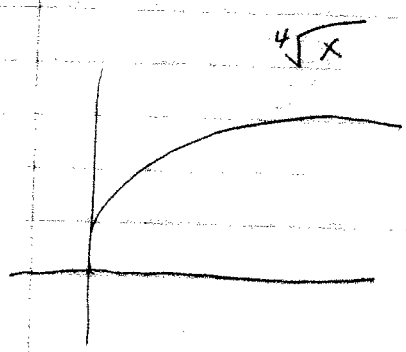
→

$x^{1/2}, x^{1/3}, x^{1/4}, \ldots$

$\sqrt{x}, \sqrt[3]{x}, \sqrt[4]{x}, \ldots$







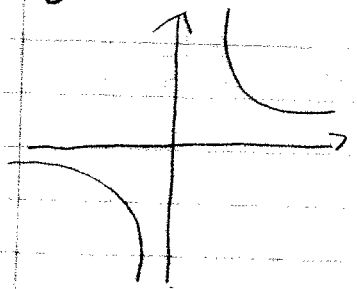
Why do these look like sideways versions of $x^2, x^3, x^4, \ldots$?

A: $y = \sqrt[3]{x} \rightsquigarrow y^3 = x$

↳ flip letters $x^3 = y$

Ex.

$g(x) = x^{-1}$ same as $\frac{1}{x}$



$(-\infty, 0) \cup (0, \infty)$] Domain + Range

$\mathbb{R} \setminus \{0\}$

② Rational Fnx

$R(x) = \frac{P(x)}{Q(x)}$ ← Polynomial fnxs

Polynomial Review

$2, -7.1, -e$ | $x^2, -2x^2 + x, -3x^2 + 1, \ldots$

4

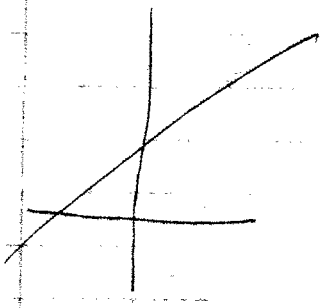
General Notation

$$a_n x^4 + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

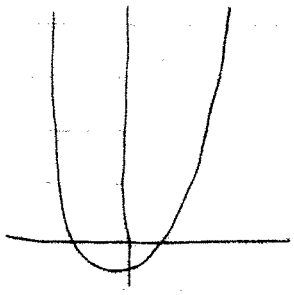
This not zero $\rightsquigarrow$ n = degree polynomial (leading term)

$a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0$ are real #'s

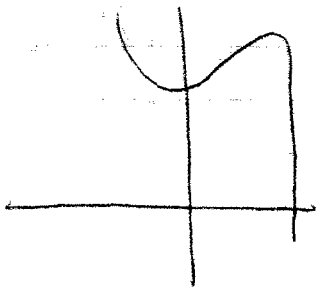
degree = 1



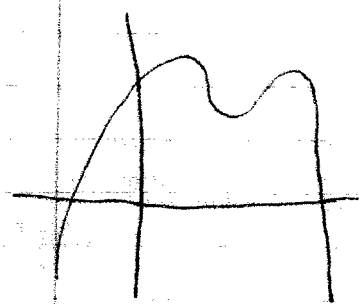
degree = 2



degree = 3



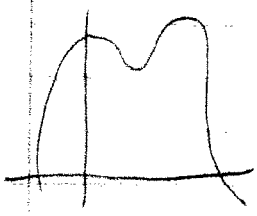
d = 4



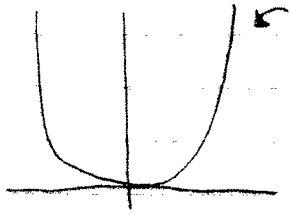
Fact: Look @ # of turns in graph. Add 1 this is smallest degree can be.

Although NOT necessarily the degree.

d = 4



x^4



also degree 4 but only 1 turn

→

③ Rational Fnx

ex.

$$\frac{x+7}{x^2+2x-3}, \quad \frac{2x^2-3}{x^{17}-.1x+7}, \ldots$$

Domain: All reals that don't make den 0

e.g. $z(t) = \frac{1}{t^2-t}$

$t^2 - t = 0$

$t = 1$ or 0

bc $t(t-1) = 0$

Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

Range: A lot of work

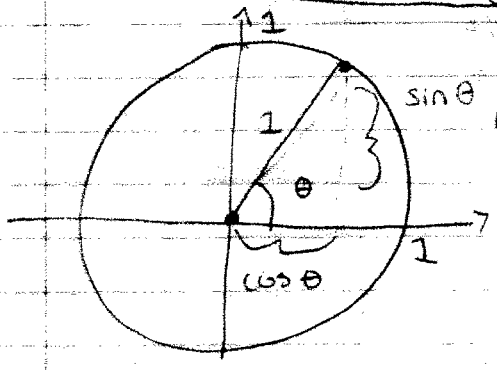
④ Algebraic Fnx

- Start w/ polynomials
- add, subtract, multiply, divide & take roots of them

$$\frac{\sqrt{x}+1}{x^3+\sqrt{x-2}(x^4+27} + \sqrt[3]{x^4+7x} + \frac{\pi x+1}{-3x^{17}+4}$$

⑤ Trigonometric Fnxs

sinx, cosx, tanx, secx, cscx, cotx



$x^2 + y^2 = 1$

$(\cos\theta, \sin\theta)$

$x \quad y$

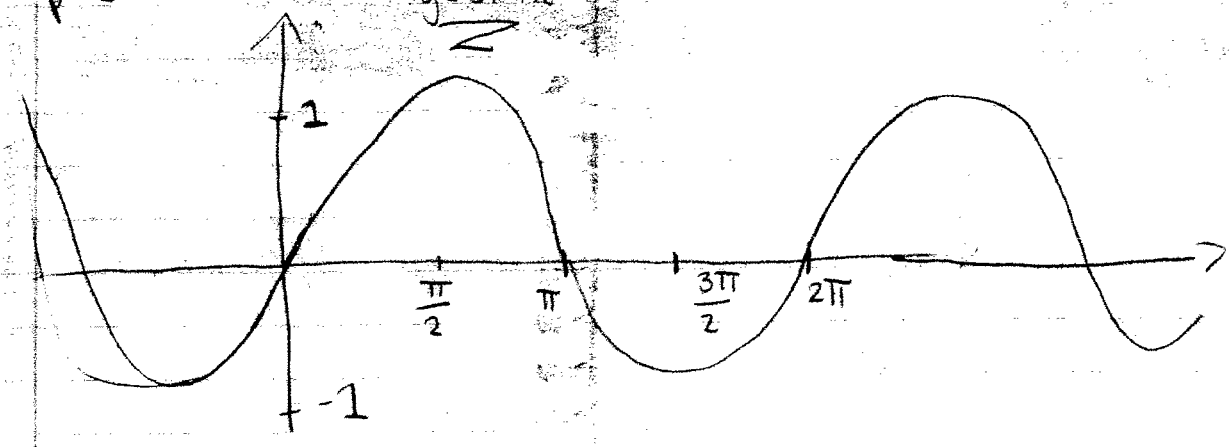
$S(\theta) = \sin\theta$

$C(\theta) = \cos\theta$

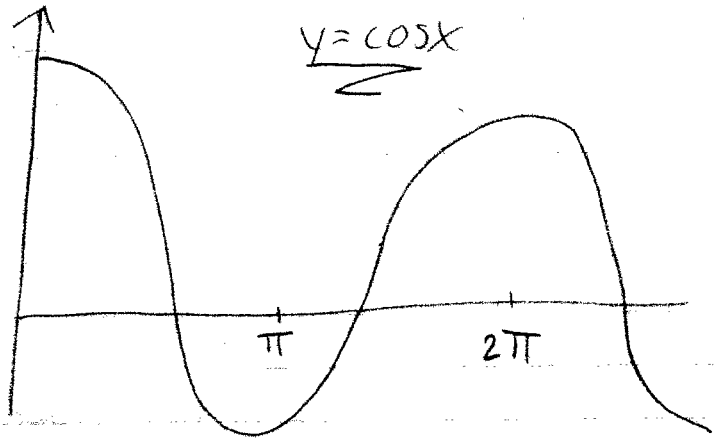
$T(\theta) = \frac{\sin\theta}{\cos\theta}$

p. 6

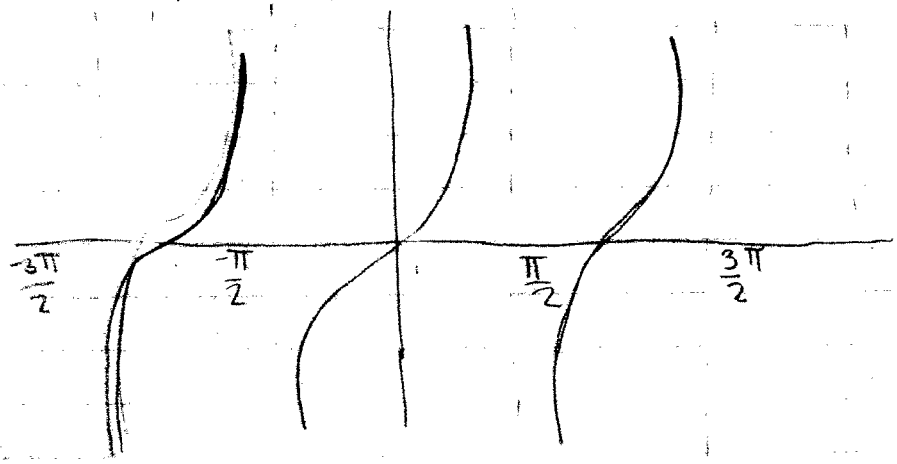
$y = \sin x$



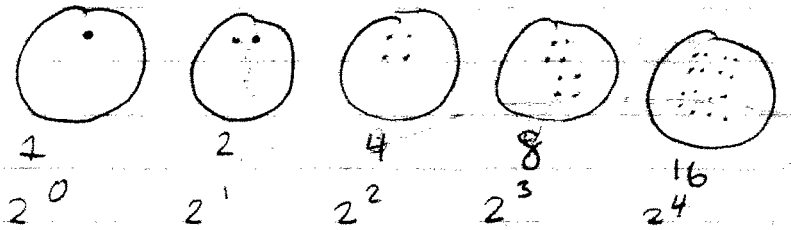
$y = \cos x$



$y = \tan x$

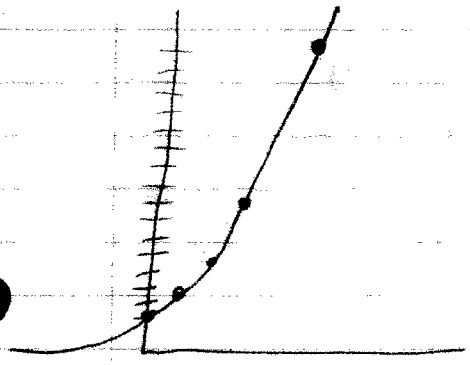


⑥ Exponential Fnx



Exponential Growth

Plot on Plane

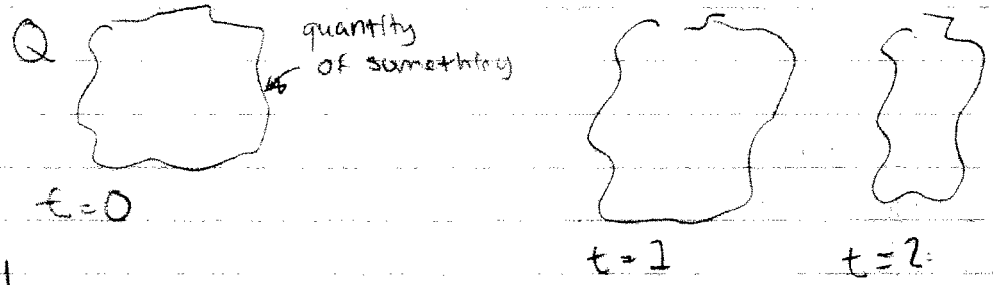


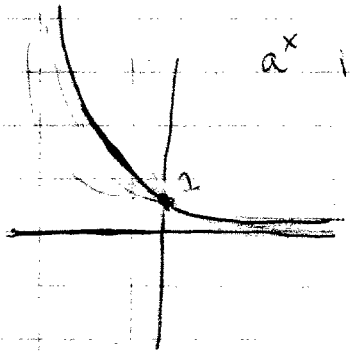
$2^x = f(x)$
exponential

Exponential Decay

a^x a is positive real #

$\frac{1}{2}Q$ $\frac{1}{2}(\frac{1}{2}Q) = \frac{1}{2}^2 Q$





a^x if $a < 1$

$a^x a^y = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

$a^0 = 1$

$a^{-1} = \frac{1}{a}$

→

(7) Logarithmic Fnxs

(Inverse fnxs to exp fnxs)

$L(x) = \log_a x$?

what power do I raise a in order to get x ?

e.g. $\log_3 27 = 3$ $\log_2 1 = 0$

$(\log_a x)$

$a^{(\log_a x)} = x$

$3^{\log_3 27} = 3^3 = 27$

$2^{\log_2 1} = 2^0 = 1$

$\log_a (a^x) = x$

Properties of Logs

$\log_a (xy) = \log_a (x) + \log_a (y)$

$\log_a (x/y) = \log_a (x) - \log_a (y)$

$\log_a (x^k) = k \log_a x$

$\log_a 1 = 0$

Problem I chose

1.1
2

37.) Find domain + sketch graph of fnx

$g(x) = \sqrt{x-5}$

$D: x \geq 5$