Alexa Vetrano 9-22-09

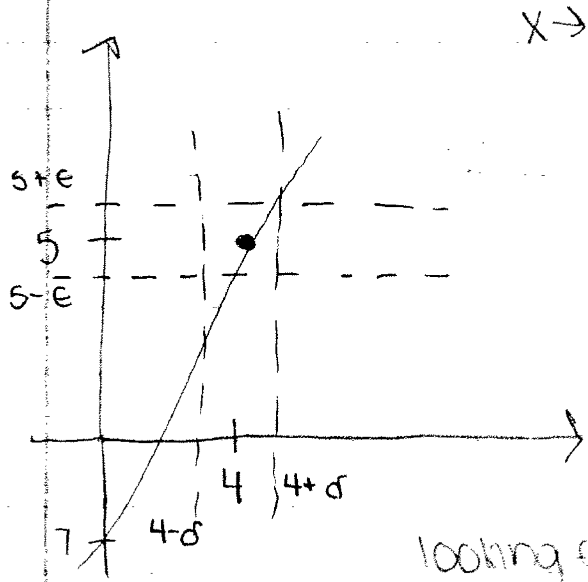
: needs to take a really long nap today!

Definition $\lim_{x \to a} f(x) = L$ if for any error (positive) $\epsilon > 0$

there is a corresponding positive number $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

If you want to prove that a function has a limit as $x \to a$ then you have to provide/find/discover $\delta > 0$ based on whatever error you're given.

ex. Prove that the limit $\lim_{x \to 4} 3x - 7 = 5$



(dont have to draw a picture)

To answer start with $\epsilon > 0$

1) Investigate and guess

we want $|f(x) - 5| < \epsilon$

$-|3x - 7 - 5| < \epsilon = |3x - 12| < \epsilon$

looking for a restriction on $|x-4|$

$3|x-4| < \epsilon \quad |x-4| < \epsilon/3 \leftarrow$ (guess) show that δ works:

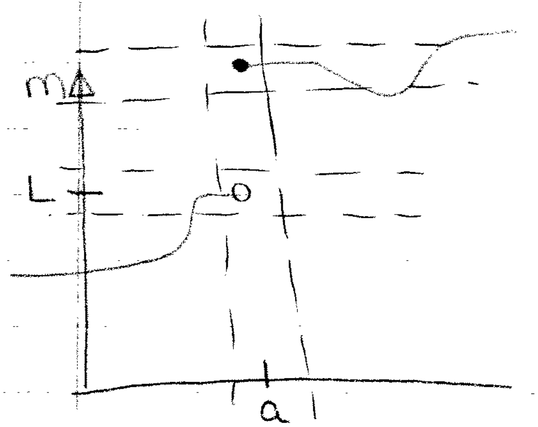
if $|x-4| < \frac{\epsilon}{3}$ then $3|x-4| < \epsilon \rightarrow |3x-12| < \epsilon$

$\rightarrow |3x-7-5| < \epsilon \rightarrow |f(x)-5| < \epsilon$

conclusion, were done :)

$$\boxed{\text{so, } \lim_{x \to 4} 3x-7=5}$$

One sided limits



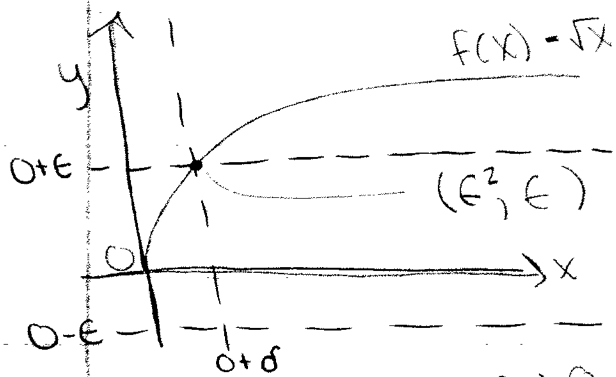
Definition 1:

$\lim_{x \to a^+} f(x) = M$ if for any $\epsilon > 0$ there is a choice of $\delta > 0$ so that if $a < x < a+\delta$ then $|f(x) - M| < \epsilon$

Definition 2:

$\lim_{x \to a^-} f(x) = L$ if for any $\epsilon > 0$ there is a choice of $\delta > 0$ so that if $a - \delta < x < a$ than $|f(x) - L| < \epsilon$

Ex. $\lim_{x \to 0^+} \sqrt{x} = 0$



give me an $\epsilon > 0$ and find $\delta > 0$

1) investigate / guess $|f(x) - 0| < \epsilon$

$|\sqrt{x} - 0| < \epsilon \rightarrow |\sqrt{x}| < \epsilon \rightarrow \sqrt{x} < \epsilon$

(Remember: we're looking for a condition on $0 < x < 0 + \delta$)

$\longrightarrow \quad x < \epsilon^2$

guess $\delta = \epsilon^2$

Show it works:

if $0 < x < \epsilon^2$

then $\sqrt{x} < \epsilon$

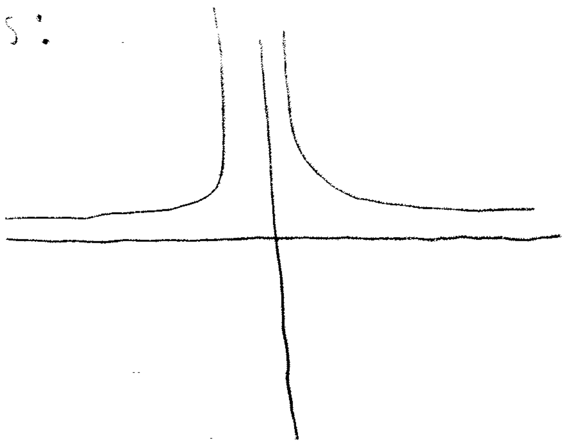
then $|\sqrt{x}| < \epsilon$

then $|\sqrt{x} - 0| < \epsilon$ so $|f(x) - 0| < \epsilon$

so $\lim_{x \to 0^+} \sqrt{x} = 0$

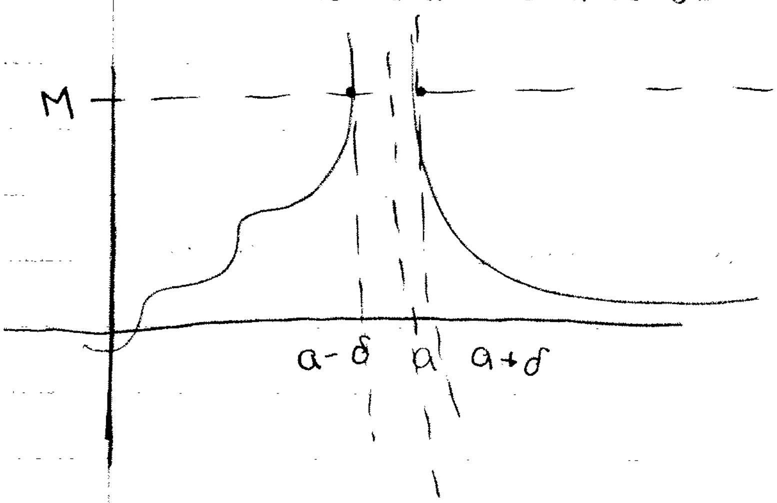
Infinite Limits:

$f(x) = \frac{1}{x^2}$



$\lim_{x \to 0} \frac{1}{x^2} = \infty$

Picture for infinite limits:



In math:

$\lim_{x \to a} f(x) = \infty$ if for every $M > 0$ there is a choice of $\delta > 0$ so that if $0 < |x - a| < \delta$ then $f(x) > M$

For any big HUGE numbere you give me M
I have to find a neigborhood of a so that every point of that neighborhood gets sent to a point on the graph higher than M

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

(1) investigate / guess:

were supplied with $M > 0$

$f(x) > M$

$\downarrow$

$\frac{1}{x^2} > M$ (we want a condition on $|x-0|$)

$1 > Mx^2$

$\downarrow$

$\frac{1}{M} > x^2 \rightarrow x^2 < \frac{1}{M} \rightarrow \underbrace{x < \frac{1}{\sqrt{M}}}_{\text{condition on } x}$

(2) show that if $0 < |x-0| < \frac{1}{\sqrt{M}}$ then $f(x) > M$