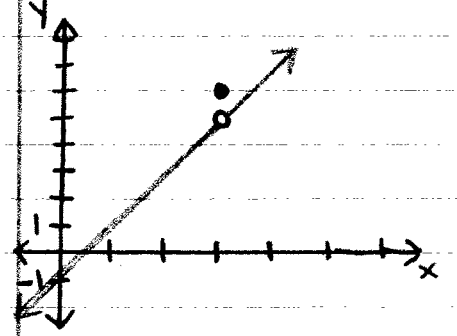
Emily F. i don't know 101. 9/21/09

Outline
- Homework: Turn in previous weeks' assignment
- Limits

Announcements
- Tuesday 9/22
BNW 254, 4-5:30
Snacks and a discussion about grad school
- FREE Tutoring details on website
- Exam Wednesday 9/30

Limits in a Minute
(Review of Homework Problem)

Limits
Concept: (by this example)

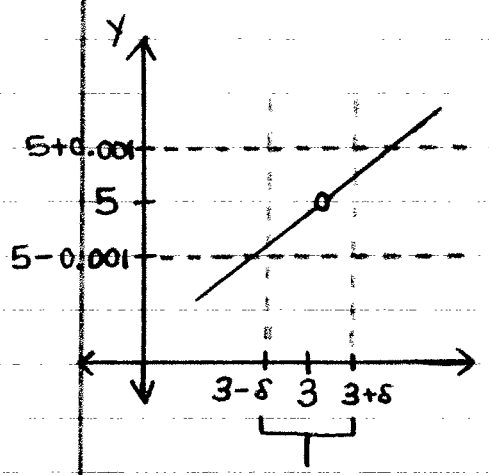


$$f(x) = \begin{cases} 2x-1 & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

Goal: Make precise the phrase "f really wants to be 5 as $x \to 3$" same as $\lim_{x \to 3} f = 5$

We asked the Question: How close does x have to be to 3 (but not equal to 3) in order for f(x) to be within 0.001 of 5?

In Math: You give me an error (0.01). and I want to find a positive number $r>0$ so that if $0<|x-3|<5$ then $|f(x)-5|<0.01$

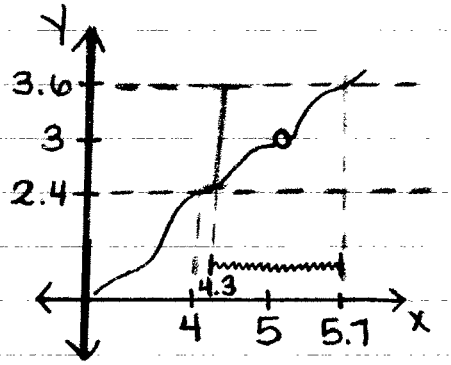


this is represented by $|f(x)-5|<0.01$

The goal is to find the δ that does this:

This is rep'd by $0<|x-3|<\delta$

Sidebar



Step 1: Investigate and Guess what we want is $|f(x)-5|<0.01$

If $x\neq 3$ then $f(x)=2x-1$

so we want

$|2x-1-5|<0.01$

$|2x-6|<0.01$

$|2(x-3)|<0.01$

$|2|\cdot|(x-3)|<0.01$

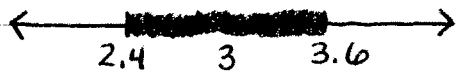
$2|x-3|<0.01$

$|x-3|<\frac{0.01}{2}$ → Guess

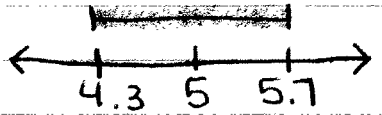
Guess: $\frac{0.01}{2}=0.005$ for δ

Q: Find the $\delta>0$ so that if $0<|x-5|<\delta$ then $|f(x)-3|<0.6$

↙ f(x) is in here



Guess: 0.7



$|x-5|<0.7$

A: $\delta=0.7$ works

Step 2: Show that Guess works

You tell me 0.01 for the error and I tell you $0.005 = \delta$ (work backwards)

If $|x-3| < 0.005$

$\rightsquigarrow 2|x-3| < 0.01$

$\rightsquigarrow |2x-6| < 0.01$

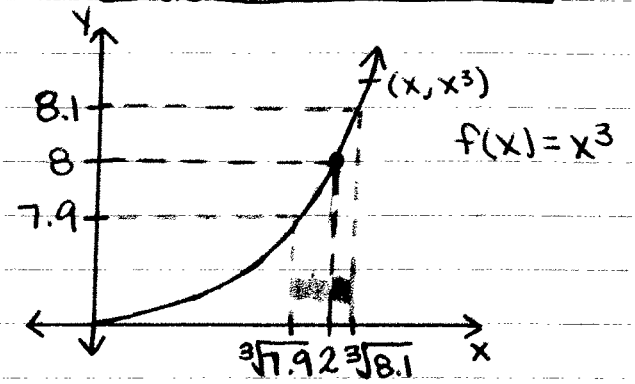
$\rightsquigarrow |2x-1-5| < 0.01$

if $x \neq 3 \rightarrow f(x)$

$|f(x)-5| < 0.01$

What δ works if the error is 0.0001?

Sidebar (continued)



Q: Find the $\delta > 0$ so that if $0 < |x-2| < \delta$ then $|f(x)-8| < 0.1$

Guess: $\delta = 0.008299$

$\sqrt[3]{7.9} = 0.0083368$

$\sqrt[3]{8.1} = 0.008299$

Find $\delta = 0$ so that if $0 < |x-3| < \delta$ then $|f(x)-5| < 0.0001$

$$\delta = 0.00005 = \frac{0.0001}{2}$$

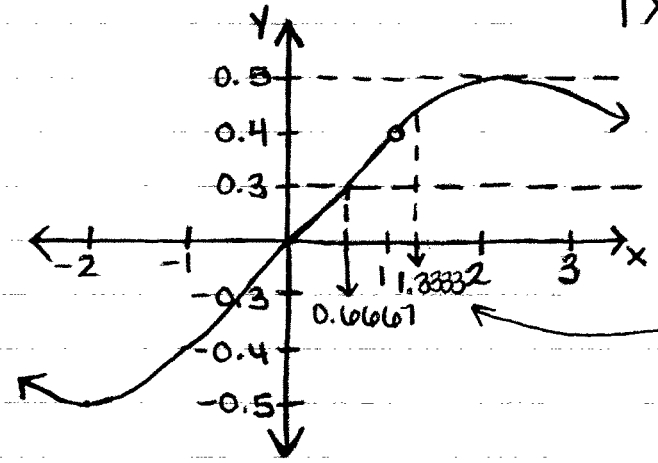
Limit of f as $x \to 3$ is 5 b/c if you give me any error $\epsilon > 0$ and I say $\delta = \frac{\epsilon}{2}$

Then this will work

Sample Problem #6

Problem: use a graph to find a number δ such that

if $|x-1| < \delta$ then $\left|\frac{2x}{x^2+4} - 0.4\right| < 0.1$



$1 + 0.3333 = 1.3333$

Start at 0.4 → y-value

$0.4 + 0.1 \rightarrow 0.5$ coordinate: $(2, 0.5)$

$0.4 - 0.1 \rightarrow 0.3$ coordinate: $(0.66667, 0.3)$

$|2-1| = 1$

$0.3333 < 1$

$|0.66667 - 1| = 0.3333$

$\boxed{\delta = 0.3333}$