9/17/09

John A. is amp'd up after taking some pictures!

Outline
limit laws (2.3)
Definition of a limit

Limit Laws (If lim f, lim g exists)

(1) $\lim (f+g) = \lim f + \lim g$

(2) $\lim fg = \lim f \lim g$

(3) $\lim f/g = \dfrac{\lim f}{\lim g}$ if $\lim g \neq 0$

(4) $\lim (c) = c$

(5) $\lim\limits_{p \to a} x^p = a^p$

(6) $\lim cf = c \lim f$

constant

Ex: use these laws to calculate
$\lim\limits_{x \to 0} \dfrac{7x^2+3}{-9x+1}$ (and justify every step)

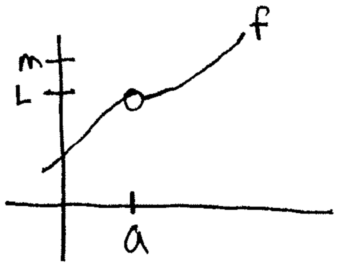
$$\lim = \frac{\lim_{x\to 0} (7x^2+3)}{\lim_{x \to 0} (-9x+1)} \ (\text{by } 3) = \frac{\lim_{x\to 0} 7x^2 + \lim_{x \to 0} 3}{\lim_{x \to 0} (-9x) + \lim 1} \ (\text{by } 1)$$

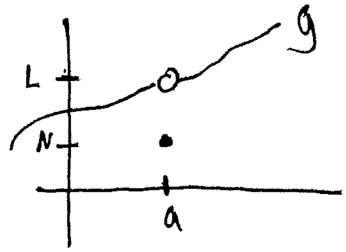
"b/c of law #3"

$$= \frac{7 \lim_{x \to 0} x^2 + 3}{-9 \lim_{x \to 0} (x) + 1} \ \left(\text{b/c of } 4,6\right) = \frac{7 \cdot 0^2 + 3}{-9 \cdot 0 + 1} \overset{(\text{by } 5)}{=} 3$$

Fact: You can evaluate the limit of a rational function or polynomial by plugging in $x = a$ as long as a is in Domain (function)

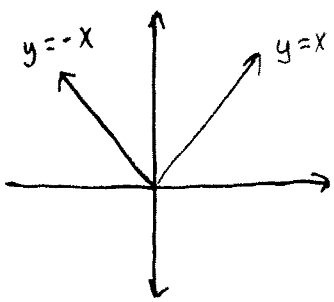
Fact: If $f=g$ everywhere (except at $x=a$) then $\lim\limits_{x\to a} f = \lim\limits_{x\to a} g$





Fact: $\lim\limits_{x\to a} f$ exists exactly when $\lim\limits_{x\to a^+} f = \lim\limits_{x\to a^-} f$

Ex: $A(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

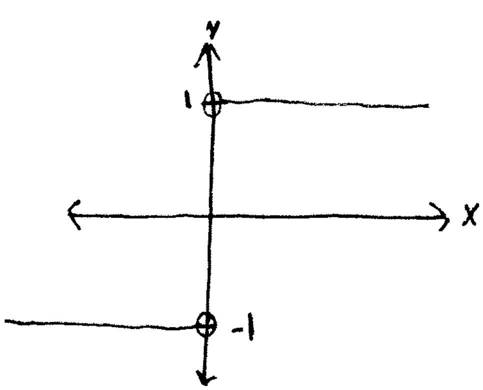


$\lim\limits_{x\to 0} |x| = 0$

b/c

$\lim\limits_{x\to 0^-} (-x) = \lim\limits_{x\to 0^+} x$

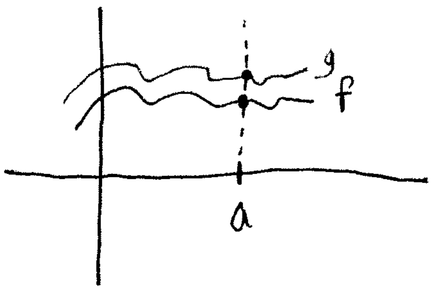
Ex: $g(x) = \dfrac{|x|}{x} = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$



$\lim\limits_{x\to 0} \dfrac{|x|}{x}$ = DNE (Does Not Exist)

$\boxed{\text{Theorm}}$: If $f \leq g$ (except maybe at a)

then $\lim_{x \to a} f \leq \lim_{x \to a} g$

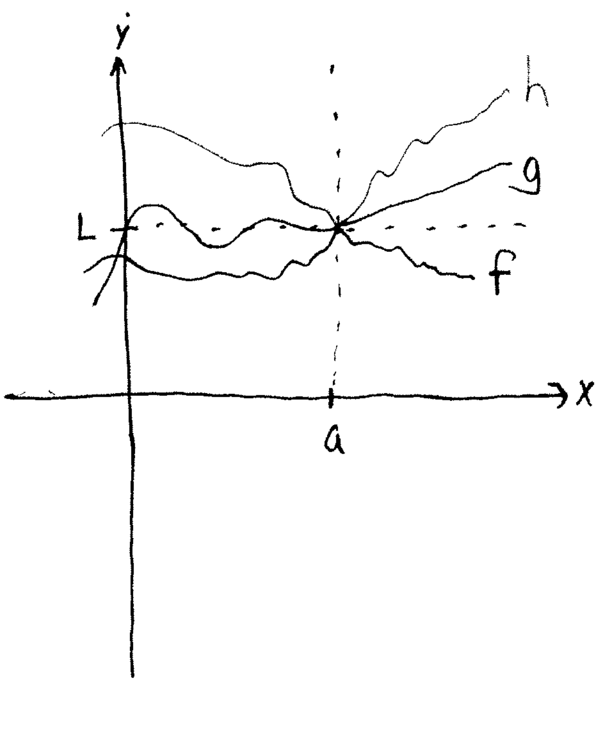


$\boxed{\text{Theorm}}$: (The "squeeze" Theorm)

If $f \leq g \leq h$ (except maybe at $x = a$)

and if $\lim_{x \to a} f = L = \lim_{x \to a} h$

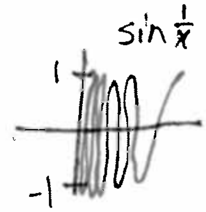
then $\lim_{x \to a} g = L$



$\boxed{\text{Ex:}}$ $f(x) = x^2 \sin\frac{1}{x}$

show $\lim\limits_{x\to 0} f = 0$

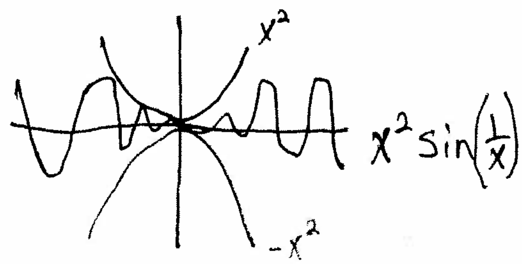
<u>solution</u>: You can't use the product limit law (b/c $\lim\limits_{x\to 0} \sin\frac{1}{x}$ DNE)



$-1 \leq \sin\frac{1}{x} \leq 1$

↓ multiply by x^2

$-x^2 \leq x^2 \sin\frac{1}{x} \leq x^2$ so $\lim\limits_{x\to 0}(-x^2) = 0 = \lim\limits_{x\to 0} x^2$



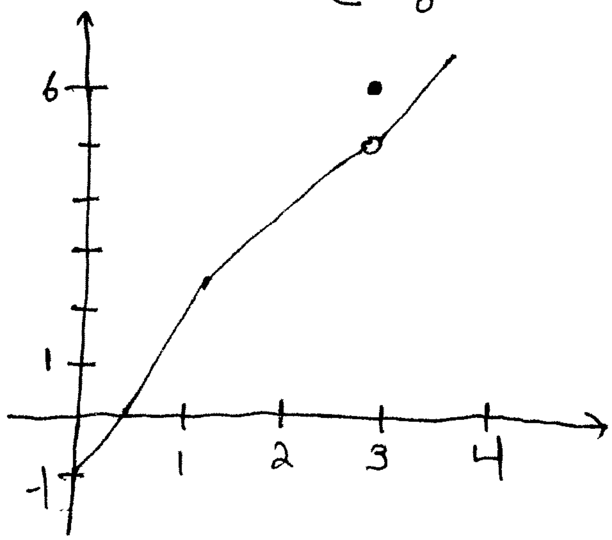
- So the limit $\lim\limits_{x\to 0} x^2 \sin\frac{1}{x} = 0$ by the squeeze law.

<u>Definitions</u> (2.4)

<u>Goal</u>: To make "x is really close to a" and "the function really wants to be L at x=a"

Ex:

$$f(x) = \begin{cases} 2x-1 & x \neq 3 \\ 6 & x = 3 \end{cases}$$



obviously our geometric intuition says $\lim_{x \to 3} f = 5$

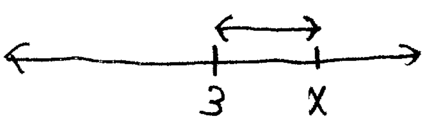
Recall our intuitive def:

- If you give me an error then I want to find a neighborhood around 3 so that every point (except maybe x=3) in that nbhd gets sent to a points that's within that error of 5.

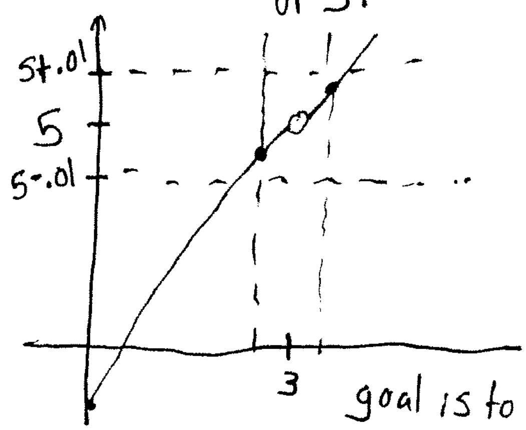
Ex: errorval

Recall:

$|x-3|$ = distance from the point x to the point 3



Note: $|x-3| = |3-x|$



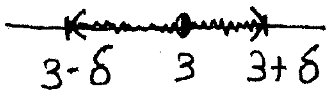
goal is to find this distance

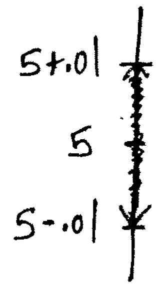
(Goal) Q: How close does x have to be to 3 in order for f(x) to be within 0.1 of 5?

In math, the goal is:

- Find a positive number (call it δ ← delta) that satisfies the following

if $\underbrace{0 < |x-3| < \delta}$, then $\underbrace{|f(x) - 5| < 0.1}$,





~~Two~~

Two steps in the process of showing the limit exists

$\boxed{\text{Step 1}}$: Guessing after you investigate
$\boxed{\text{Step 2}}$: show your guess works

Step 1:

We want $|f(x) - 5| < 0.1$ // as long as $x \neq 3$

$$|(2x-1)-5| = |2x-6| = |2(x-3)| = 2|x-3|$$

So we want $2|x-3| < 0.1$

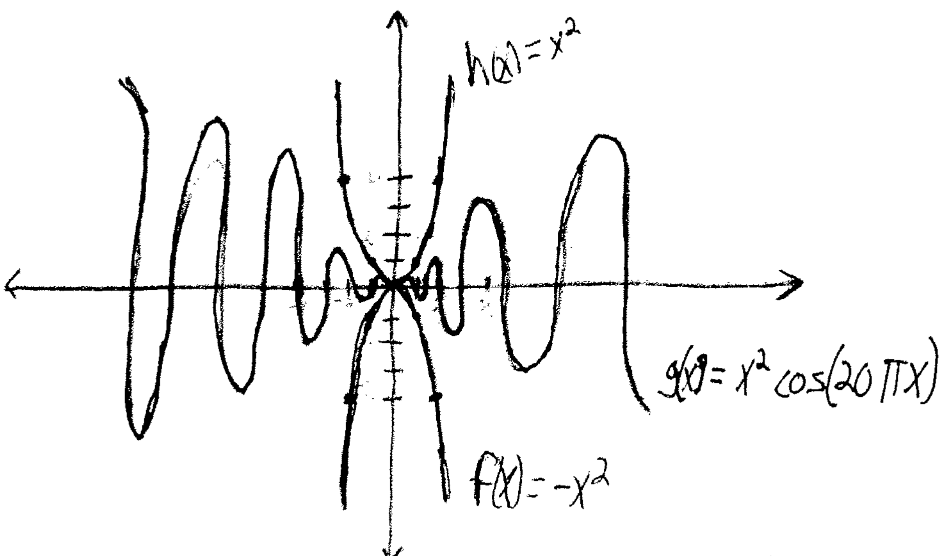
$$|x-3| < \frac{0.1}{2}$$

Johns Class notes example problem

"Squeeze Theorm"

Page 85, problem # 33

- Use squeeze theorm to show that $\lim_{x\to 0} (x^2 \cos 20\pi x) = 0$
  - Illustrate by graphing the functions: $f(x) = -x^2$
    $g(x) = x^2 \cos 20\pi x$
    $h(x) = x^2$



- $f(x) \leq g(x) \leq h(x)$

$\rightarrow$ **So**: $f(x) = h(x)$

$\rightarrow -x^2 = x^2 \rightarrow 0 = 2x^2 \rightarrow x = 0$

If $x=0$, then $\{f(x) = 0 \leq g(x) \leq h(x) = 0\}$ (do this by plugging 0 for x)

$\boxed{\text{So}}$: $g(x) = x^2 \cos 20\pi x = 0$
when the limit of $x \rightarrow 0$.

*(note the limit $x \rightarrow 0$ of $f(x)$, and $h(x)$ is also 0)