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Outline

* HW + Quiz solutions are posted on website, just the solutions to what I decided to grade *

- Quiz
- HW problem
- Limit Laws § 2.3

About the HW:

Collaboration Rule: Work together, but tell me on HW who you worked with.

Announcements

- Tomorrow c/o picture
- 9/22 @ 4pm Graduate Sch in Math Inf Panel

Limit Laws (§ 2.3)

Suppose $\lim_{x \rightarrow a} f = L$ & $\lim_{x \rightarrow a} g = M$

① $\lim_{x \rightarrow a} (f+g) = \lim_{x \rightarrow a} f + \lim_{x \rightarrow a} g = L+M$

② $\lim_{x \rightarrow a} (f-g) = \lim_{x \rightarrow a} f - \lim_{x \rightarrow a} g = L-M$

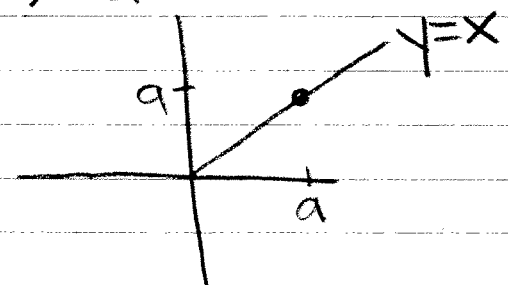
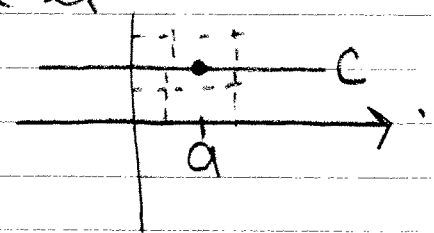
③ $\lim_{x \rightarrow a} (fg) = L \cdot M$

④ $\lim_{x \rightarrow a} (f/g) = L/M$ as long as $M \neq 0$

⑤ $\lim_{x \rightarrow a} c f(x) = c L$ (c is any constant)

⑥ $\lim_{x \rightarrow a} c = c$

⑦ $\lim_{x \rightarrow a} x = a$



$$\textcircled{8} \lim_{x \rightarrow a} x^n = a^n$$

$$\textcircled{9} \lim_{x \rightarrow a} x^{1/n} = a^{1/n}$$

$$\textcircled{10} \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f}$$

Examples

$$\rightarrow 2x^2 - 3x + 4 = f(x)$$

$$\lim_{x \rightarrow 5} f(x) = \lim_{x \rightarrow 5} (2x^2 - 3x + 4)$$

$$\text{Rule 1: } \lim_{x \rightarrow 5} = \lim_{x \rightarrow 5} 2x^2 - \lim_{x \rightarrow 5} 3x + \lim_{x \rightarrow 5} 4$$

$$= 2 \lim_{x \rightarrow 5} x^2 - 3 \lim_{x \rightarrow 5} x + 4$$

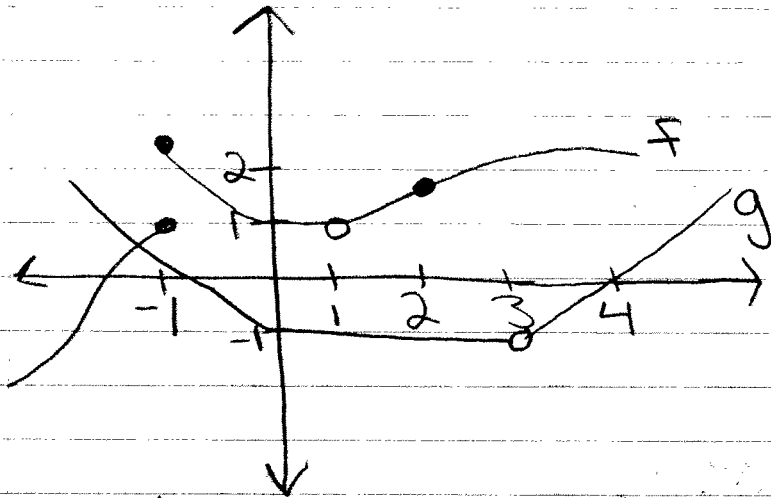
$$= 2 \cdot 5^2 - 3 \cdot 5 + 4$$

$$= 50 - 15 + 4$$

Theorem \rightarrow The limit of a polynomial or a rational function as $x \rightarrow a$ is gotten by plugging in a , AS LONG AS a is in the domain of the function.

$$\text{Dom} \left(\frac{x^2 + 2x - 3}{5x + 3} \right) = \text{All Reals except } \frac{-3}{5}$$

Ex. $\lim_{x \rightarrow a} \frac{x^2 + 2x - 3}{5x + 3} = \frac{a + 2a - 3}{5a + 3}$
 as long as $a \neq -3/5$



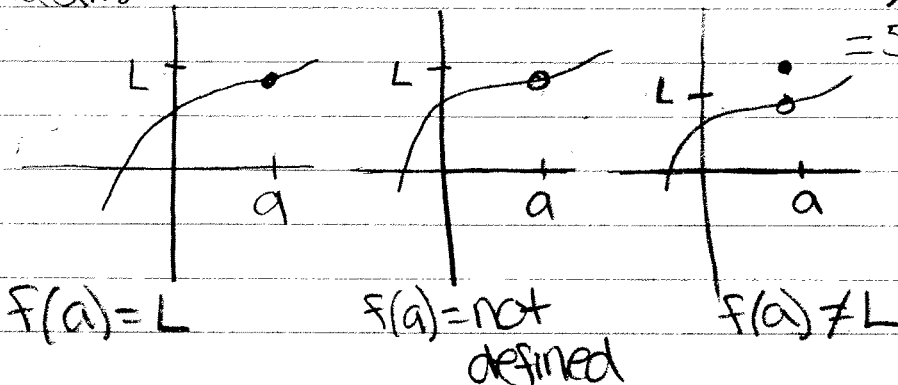
$\lim_{x \rightarrow 0} (f+g) = 1 + (-1)$
 $\lim_{x \rightarrow 4} f \cdot g = \text{something}$
 $= 0$

$\lim_{x \rightarrow 2} 5f - 2g$
 $= 5 \cdot 2 - 2(-1) = 12$

Question
 What if there is a defined value for f at $x=1$, say $f(1)=3$?

$\lim_{x \rightarrow 1} 5f - 2g$
 $= 5 \lim_{x \rightarrow 1} f - 2 \lim_{x \rightarrow 1} g$
 $= 5 \cdot 3 - 2(-1) = 17$

Recall:



Answer

The limit of f at $x=1$ is still 1
 so $\lim_{x \rightarrow 1} 5f - 2g = 5(1) - 2(-1) = 7$

$\lim_{x \rightarrow -1} f/g =$ can't use our laws to do this b/c $\lim_{x \rightarrow -1} g = 0$

$\lim_{x \rightarrow -1} g/f =$ can't answer using limit laws b/c the \lim of f as $x \rightarrow -1$ D.N.E

$$\text{But } \lim_{x \rightarrow -1^-} g/f = \frac{0}{1} = 0$$

$$\lim_{x \rightarrow -1^+} g/f = \frac{0}{2} = 0$$

My Example:

Evaluate the limit, if it exists.

$$\begin{aligned} \text{17. } \lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} &= \frac{(4+h)(4+h) - 16}{h} \\ &= \frac{16 + 8h + h^2 - 16}{h} \\ &= \frac{8h + h^2}{h} \\ &= \frac{h(-8 + h)}{h} \\ &= 8 + h \\ &= 8 + 0 \\ &= 8 \end{aligned}$$

Steps to complete

- 1) Foil out what is in the parenthesis
- 2) Notice what you can cancel.
- 3) Rewrite the problem
- 4) Factor out if you can.
- 5) Cancel out
- 6) Plug H in
- 7) Add the final answer!