Peter C. loves quizes that don't mean much

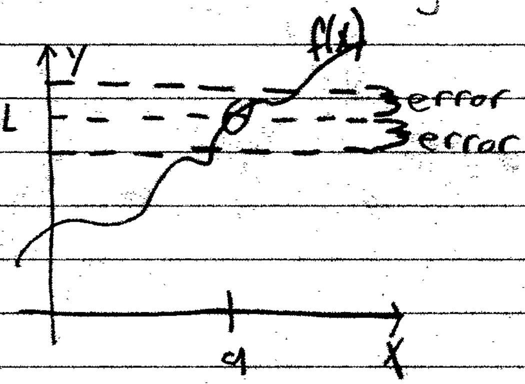
Limits

$\lim_{x \to a} f(x) = L$

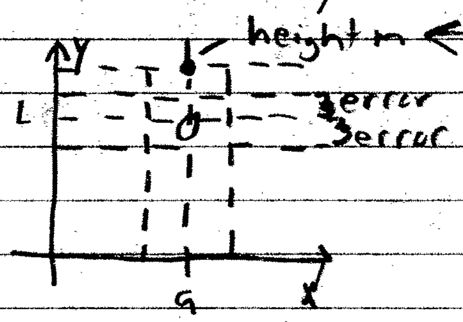
Outline
- Limits
- Quiz

"the limit as x goes to a of f is L."

Announcement
- Shape of Space Tues. Oct 6 5-6
- Exam-Wed. 9/3



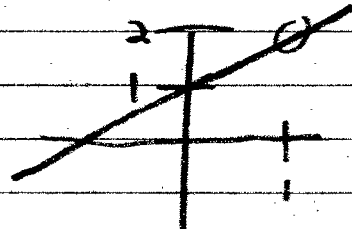
In words, in order for f to have the limit L as $x \to a$ there has to be a "neighborhood" around "a" satisfying the fact that every point (except maybe a) in that neighborhood gets mapped to within any error from L that we give



$f(a) = M$

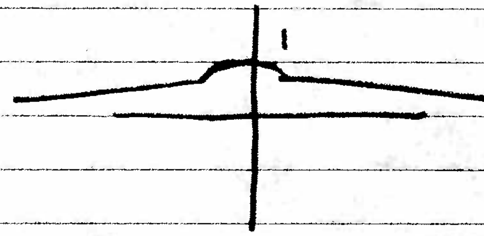
but $\lim_{x \to a} F = L$

Ex: $\frac{x^2-1}{x-1} = f(x)$



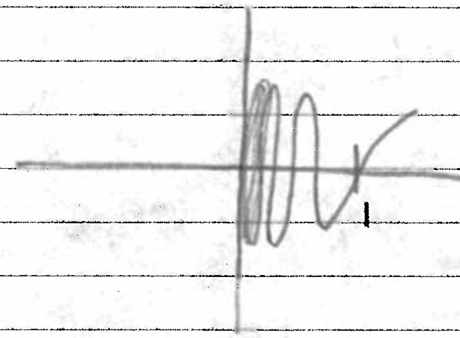
$\lim_{x \to 1} f = 2$ b/c $\frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x+1$

$f(x) = \frac{\sin x}{x}$



$f(0) = \text{undefined}$

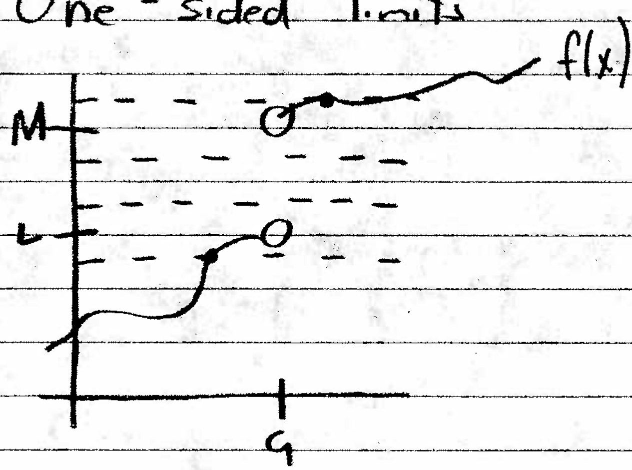
$G(t) = \sin(\pi/t)$



$\lim_{t \to 0} G(t) = \emptyset$

t	π/t
1	π
1/2	2π
1/3	3π
1/4	4π
1/100	100π

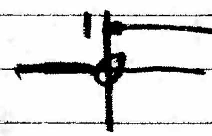
One-sided limits



$\lim_{x \to a^-} f = L$ (as x goes to a "from the left")

$\lim_{x \to a^+} f = M$ (as x goes to a "from the right")

Ex



$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 1 \end{cases}$

$\lim_{x \to 0} H = \emptyset$ but $\lim_{x \to 0^-} H = 0$

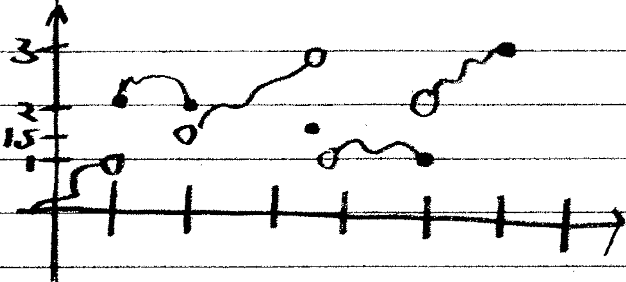
$\lim_{x \to 0^+} H = 1$

Fact

$\lim_{x \to a} f = L$

$\updownarrow$

$\lim_{x \to a^-} f = L = \lim_{x \to a^+} f$



Exercises

$\lim_{x \to 1} f = \emptyset$

$\lim_{x \to 1^-} f = 1$

$\lim_{x \to 1} f = 2$

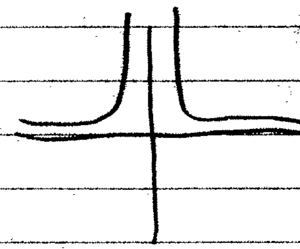
$f(3.8) = 1.5$

$\lim_{x \to 3.8} f = \emptyset$

$\lim_{x \to 3.8^+} f = 1$

$\lim_{x \to 3.8^-} f = 2.9$

$g(x) = \frac{1}{x^2}$

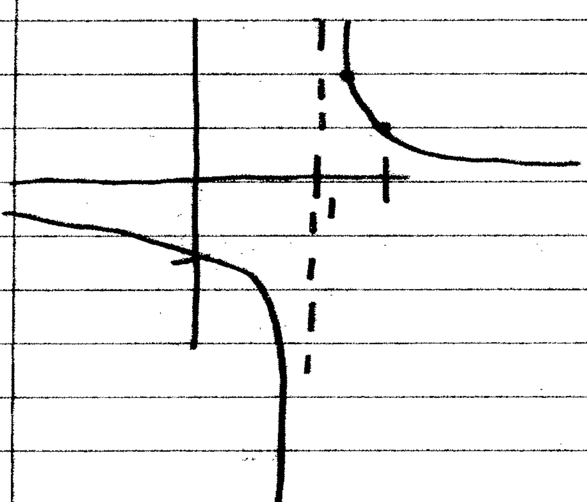


$\lim_{x \to 0} g = ?$

$\lim_{x \to 0} g = \infty$

the limit as x approaches 0 is ∞

ex: $\frac{1}{x} - 1 = f(x)$

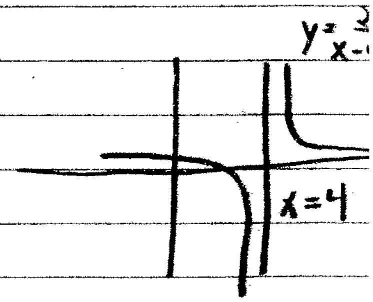


$\lim_{x \to 1} f = \emptyset$ b/c lim is different from opposite side

because f has different behavior on either side of 1

$\lim_{x \to 1^+} f = \infty$ $\lim_{x \to 1^-} f = -\infty$

Find $\lim_{x \to 4^+} \frac{2x}{x-4}$ & $\lim_{x \to 4^-} \frac{2x}{x-4}$



$\lim_{x \to 4^+} \frac{2x}{x-4} = -\infty$; $\lim_{x \to 4^+} \frac{2x}{x-4} = \infty$