Iryna Mohylnytska
09/10/11

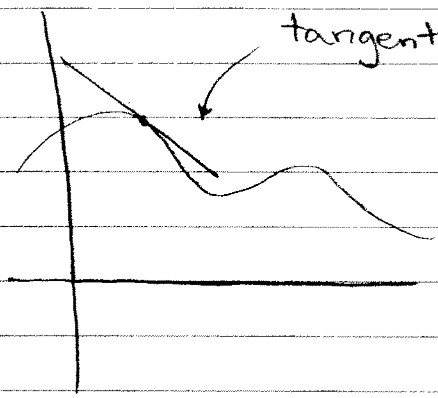
Outline

- Tangents / Velocity
- Limits

Announcements

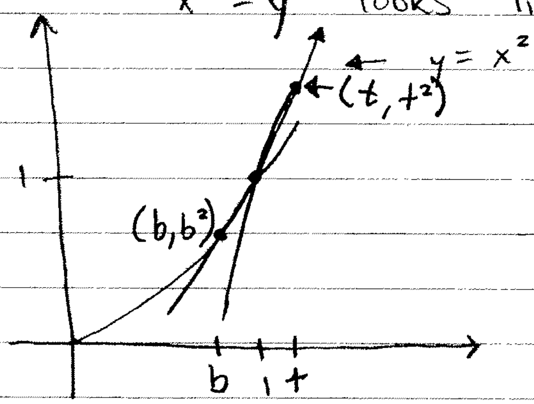
- Quiz Monday

Recall: Tangents to curves



tangents have a point at which they rest "nicely" on a curve.

Example: Let's try to guess what the tangent line to $x^2 = y$ looks like at $(1, 1^2)$



slope $\frac{t^2-1}{t-1}$ $\frac{\text{rise}}{\text{run}}$

$= t+1$

slope $\frac{1-b^2}{1-b} = 1+b$

The slope of the line from $(1,1)$ to (t, t^2) is very close to 2 if t is very close to 1.
(and the same thing for the line thru $(1,1)$ & (b, b^2))

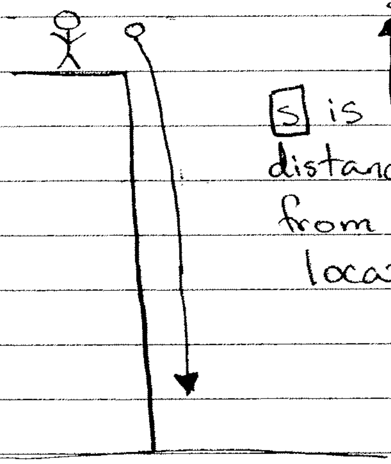
Guess: The slope of the tangent line is 2 at $(1,1)$
(turns out it's correct).

Velocity

$s(t) = 49t^2$

$\boxed{s}$ is distance in meters from the drop location

$\boxed{t}$ is time from letting the obje go.



Example: Guess how fast an object is traveling at 5 seconds.

$$\text{Velocity} = \frac{\text{How far}}{\text{How long}} = \frac{\text{distance}}{\text{time}} \rightarrow \frac{\text{meters}}{\text{seconds}}$$

velocity is m/s

Time interval 0.1 secs

$$\frac{\text{how far}}{\text{how long}} = \frac{s(5.1) - s(5)}{5.1 - 5} = \frac{4.9(5.1)^2 - 4.9(5)^2}{0.1} = \boxed{49.49\ \frac{m}{s}}$$

In words, the average speed from time 5 to see to time 5.1 sec is 49.49 m/s.

Time Interval 0.01

$$\frac{\text{how far}}{\text{how long}} = \frac{s(5.01) - s(5)}{0.01} = 49.049\ m/s$$

Homework
Section 2.1: #1, 6

Time interval "h"

$$\frac{s(5+h) - s(5)}{h} = \frac{4.9(5+h)^2 - 4.9(25)}{h} =$$

$$\frac{4.9(25+10h+h^2) - 4.9(25)}{h} = \frac{49h + 4.9h^2}{h}$$

$$= \underbrace{49 + 4.9h}$$

Note: This is exactly what we did when we calculated the slope of the tangent line in the last example

Guess: the velocity at 5 sec. at the falling object 49 m/s (instantaneous velocity)

Limits:

Consider the function

$f(x) = x^2 - x + 2$

What's the behavior of f around $x=0$?

Prof. R then went to the overhead to examine the graph of f and set up a table of function values

Behavior: Around the value $x=0$, the function seems to want to take the value 2.

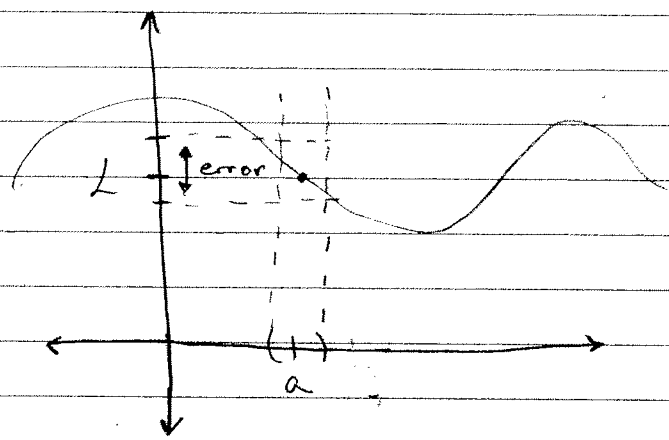
Notation/Definition:

$$\lim_{x \to 0} f(x) = 2$$

Say "the limit of the function f as x approaches 0 is 2"

Sometimes we write "$f \to 2$ as $x \to 0$"

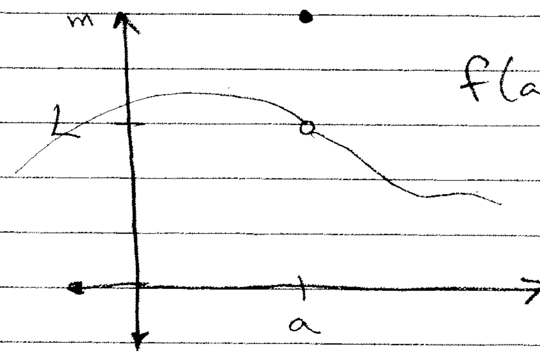
Geometrically



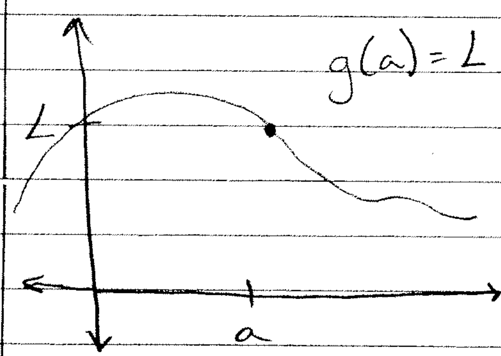
The function has limit L if the points around a on the x-axis get sent close to L (with a prescribed error).

Geometric examples illustrating this

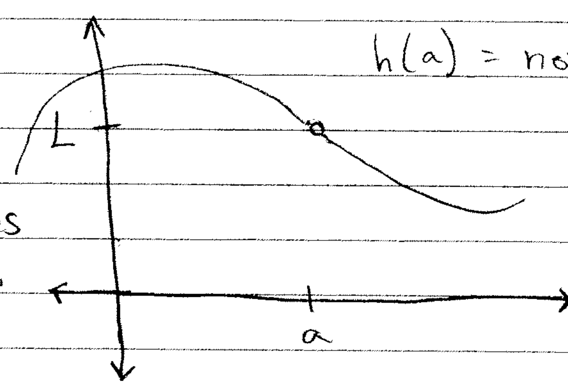
$f(a) = m$



$g(a) = L$



$h(a) =$ not defined



In all of these examples the limit as the x-values approach a is L.

$\lim_{x \to a} f = L \qquad \lim_{x \to a} g = L \qquad \lim_{x \to a} h = L$

Prof. R. went back to the graphing calculator to show more examples of limits:

$f(x) = \dfrac{\sqrt{x+9} - 3}{x^2} \qquad g(x) = \dfrac{\sin x}{x} \qquad h(x) = \sin\left(\dfrac{\pi}{x}\right)$

Warning: looking at a table to guess a limit doesn't always work.

Example: $\sin\left(\frac{\pi}{x}\right)$

$\underbrace{h(x) = \sin(\pi/x)}$
this limit
as $x \to 0$ is NOT 0!

x	$\sin \pi/x$
$1/2$	$\sin(2\pi) = 0$
$1/3$	$\sin(3\pi) = 0$
$1/4$	$\sin(4\pi) = 0$
$1/5$	0
$1/6$	0

Scribe problem #5 pg. 75

Use the graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

a) $\lim_{x \to 1^-} f(x) = 2$

b) $\lim_{x \to 1^+} f(x) = 3$

c) $\lim_{x \to 1} f(x)$ = DNE because the limit as $x \to 1^+$ and as $x \to 1^-$ is not the same therefore the limit does not exist as $x \to 1$.

d) $\lim_{x \to 5} f(x) = 4$

e) $f(5)$ = DNE, because there's a hole at that point.