Kaila Anderson 10/8/09

<u>outline</u>

- Trig derivatives
- Chain rules

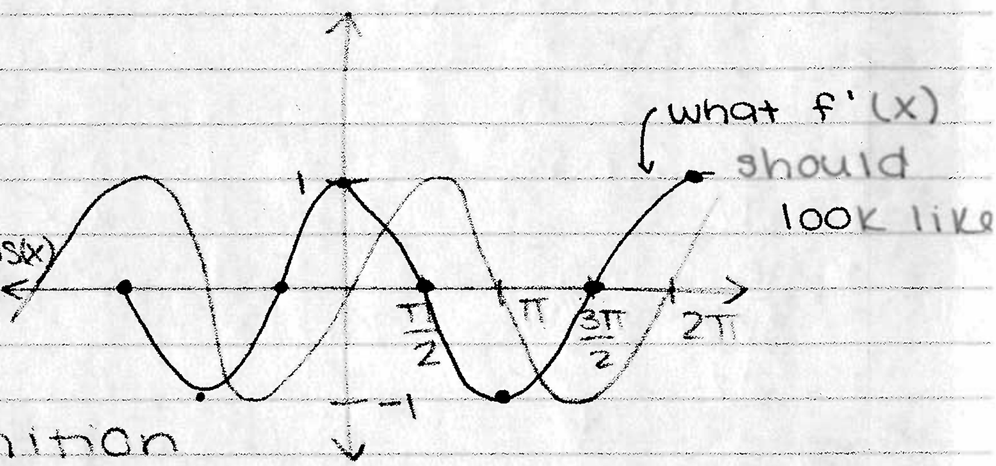
<u>Announcement</u>

- Tuesday @ 9:30 class

Trigonometric Functions

$f(x) = \sin(x)$

<u>Fact</u>: $\frac{d}{dx}\sin(x) = \cos(x)$

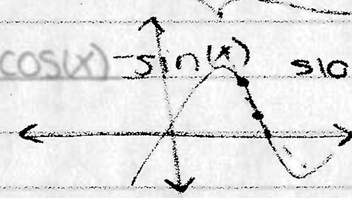


using the definition of the derivative

$$\frac{d(\sin(x))}{dx} = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

using a trig identity

$$\lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$



slope of secant is $\frac{\sin(x+h) - \sin(x)}{h}$

$$= \lim_{h\to 0} \frac{\sin(x)[\cos(h) - 1] + \sin(h)\cos x}{h}$$

$$= \lim_{h\to 0} \boxed{\sin(x)} \frac{(\cos(h) - 1)}{h} + \lim_{h\to 0} \frac{\sin(h)}{h} \cdot \boxed{\cos x}$$

$\sin(x)$ + $\cos(x)$ don't depend on h

- so we can pull them from the limit

$$\boxed{\sin(x)} \lim_{h\to 0} \boxed{\frac{\cos(h)-1}{h}} + \boxed{\cos(x)} \lim_{h\to 0} \boxed{\frac{\sin(h)}{h}}$$

Facts:

$$\lim_{h\to 0} \frac{\sin(h)}{h} = 1$$

$$\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$$

Terminology:
Anytime we take a limit where plugging in would give us $\frac{0}{0}$ its called an indeterminate form

so $\frac{d}{dx}\sin(x) = \cos(x)$

Rules

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{(\cos(x))^2}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \frac{1}{\cos x}\cdot\frac{1}{\cos x} =$$

$$\sec x \cdot \sec x = \sec^2 x$$

Recall: $\sec x \overset{\text{def}}{=} \frac{1}{\cos x}$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

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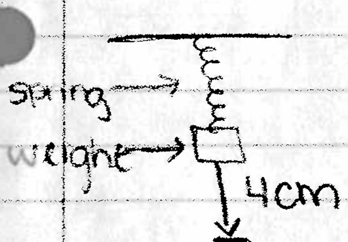
$\frac{d}{dx}\sec x = \sec x \cdot \tan x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}\cot x = -\csc^2 x$

Ex. $f(x) = x^2 \sin x$ (use product rule) $x^2 \cdot \sin$
$f'(x) = x^2 \cos x + 2x \sin x$

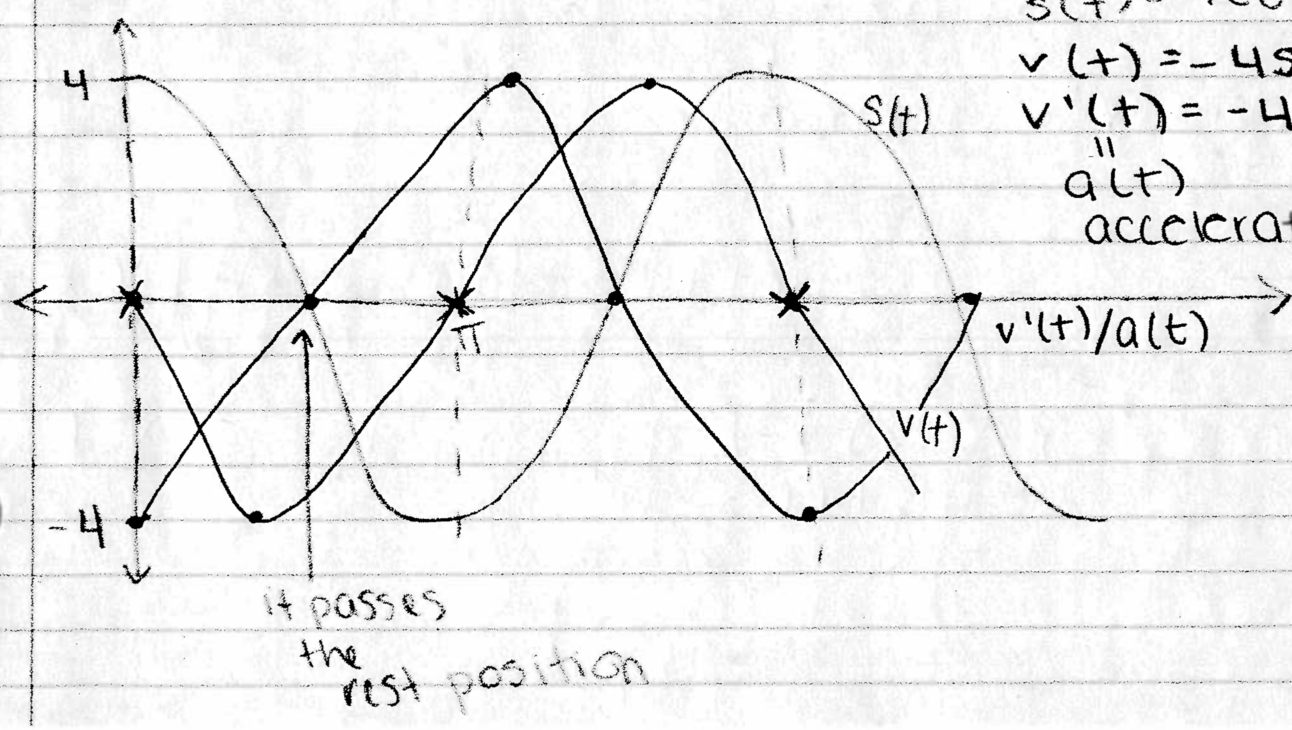
Ex.



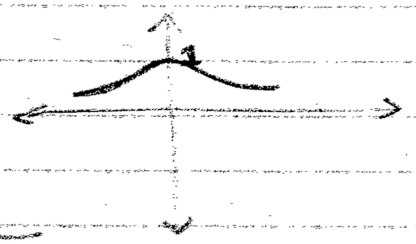
suppose the distance from the resting position at time t seconds is
$s(t) = 4\cos t$ cm
(notice $s(0) = 4\cos 0 = 4$)

Let's analyze $v(t)$ and $v'(t)$ in this situation

$s(t) = 4\cos t$
$v(t) = -4\sin t$
$v'(t) = -4\cos t$
$= a(t)$
acceleration



$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



use this fact
to calculate limits
that are new

Ex. $\lim_{x \to 0} \frac{\sin 7x}{4x} = ?$

Here's a trick: $\dfrac{\sin 7x}{\frac{7}{4} \cdot 4x} = \dfrac{\sin 7x}{7x}$

analyze $\lim_{x \to 0} \frac{\sin 7x}{7x} = 1 \to \frac{\sin 7x}{4x} = \frac{7}{4}\left(\frac{\sin 7x}{7x}\right)$

$\frac{\sin(\sim)}{\sim} \to 1$
as $\sim \to 0$

The original question:

$$\lim_{x \to 0} \frac{\sin 7x}{4x} = \lim_{x \to 0} \frac{7}{4} \frac{\sin 7x}{4x} = \frac{7}{4} \lim_{x \to 0} \overset{\nearrow 1}{\frac{\sin 7x}{7x}} =$$

Problem from textbook:
3.4

10) $y = \frac{1 + \sin x}{x + \cos x}$ quotient rule $\to \dfrac{[g(x) \cdot f'(x)] - [f(x) \cdot g'(x)]}{(g(x))^2}$

$$\frac{[(x + \cos x)(\cos x)] - [(1 + \sin x)(1 - \sin x)]}{(x + \cos x)^2}$$