

Alessia (aise)

10/7/09

Outline

stuff you already know!  
(Derivatives)

- Quiz Tomorrow  
on Derivatives (covers the previous  
week + today)

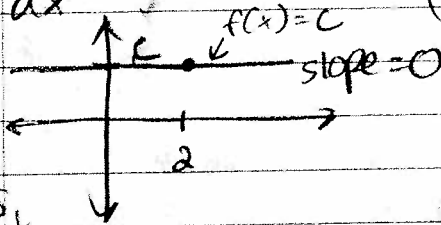
Differentiation Formulas

①  $\frac{d}{dx}(c) = 0$

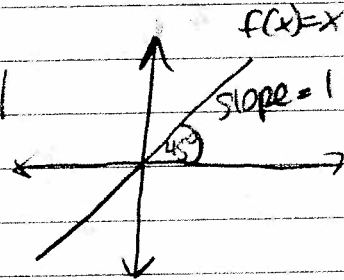
(why?  
using limits

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} 0 = 0$



②  $\frac{d}{dx}(x) = 1$



(using the derivative definition  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ )  
 $\lim_{h \rightarrow 0} \frac{x+h - x}{h} = \frac{h}{h} = 1$

exercise Show  $f'(x) = 2$   
if  $f(x) = 2x - 3$

geometrically and using the  
definition of the derivative

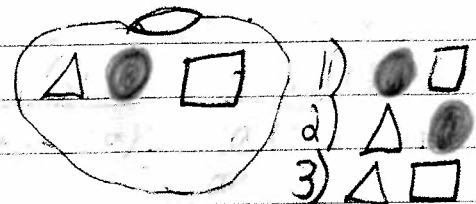
③  $\frac{d}{dz}(z^n) = nz^{n-1}$

sidebar:

1	1	1	$(a+b)^0 = 1$				
1	2	1	$(a+b)^1 = a + b$				
1	3	3	1	$(a+b)^2 = a^2 + 2ab + b^2$			
1	4	6	4	1	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$		
1	5	10	10	5	1	$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	
1	6	15	20	15	6	1	

exercise

Show  $\frac{d}{dx} nx^{n-1}$  use it  $(x+h)^n$   
with



constant

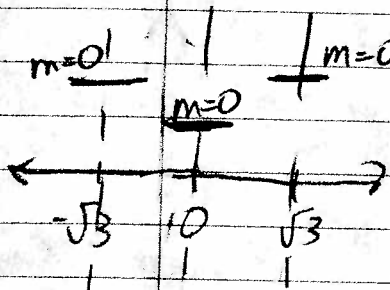
$$(4) \frac{d}{dy} (c f(y)) = c \frac{d}{dy} (f(y)) \quad \text{ex) } f(x) = 2x^4$$

$$f'(x) = 8x^3$$

$$(5) \frac{d}{dx} (f(x) \pm g(x)) = f'(x) \pm g'(x)$$

using 1-5 ex)  $f(x) = 17x^2 - 2x^7 + 3x - 12$

$$34x - 14x^6 + 3$$



ex)  $y = x^4 - 6x^2 + 4$  Find the points on this curve with horizontal tangent lines

$$y' = 4x^3 - 12x$$

$$0 = 4x^3 - 12x$$

$$4x(x^2 - 3)$$

$$x = 0 \quad | \quad x = \pm\sqrt{3}$$

So this curve has horizontal tangents at  $x = 0, \pm\sqrt{3}$

$$(6) \frac{d}{dx} (f(x) \cdot g(x)) = f(x)g'(x) + f'(x)g(x)$$

$$(7) \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

ex)  $\frac{x^4 + 3x - 1}{2x^5 - 2x^2} = f(x)$   $F'(x) = \frac{(4x^3 + 2)(2x^5 - 2x^2) - (x^4 + 3x - 1)(10x^4 - 4)}{(2x^5 - 2x^2)^2}$

Let It Like This!

ex)  $f(x) = \frac{1}{x}$   $f'(x) = x^{-1} = -x^{-2}$

ex)  $g(z) = z^n$   $g'(z) = n z^{n-1}$

~~(ex)  $h(x) = 7^x$   $h'(x) = x 7^{x-1}$~~

warning Don't confuse the power function with the exponential function

$$\textcircled{53} \quad y = x + \sqrt{x} \quad (1, 2)$$

tangent line

$$x + (x)^{1/2} \\ 1 + \frac{1}{2}x^{-1/2} = \frac{2\sqrt{x} + 1}{2\sqrt{x}} = \frac{2\sqrt{x} + 1}{2\sqrt{x}} = \frac{2\sqrt{1} + 1}{2\sqrt{1}} = \frac{2 + 1}{2} = \frac{3}{2}$$

$$m = \frac{3}{2} \quad y = mx + b \quad 2 = \frac{3}{2}(1) + b$$

$$\frac{(2)}{2} - \frac{3}{2} = b \quad \frac{4-3}{2} = b = \frac{1}{2} \quad \boxed{y = \frac{3}{2}x + \frac{1}{2}}$$

normal line  $m = -\frac{2}{3} \quad y = mx + b \quad 2 = -\frac{2}{3}(1) + b$

$$\boxed{y = -\frac{2}{3}x + \frac{8}{3}}$$

$$\frac{(3)}{3}2 + \frac{2}{3} = b \quad \frac{6+2}{3} = \frac{8}{3} = b$$

