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10/22/09

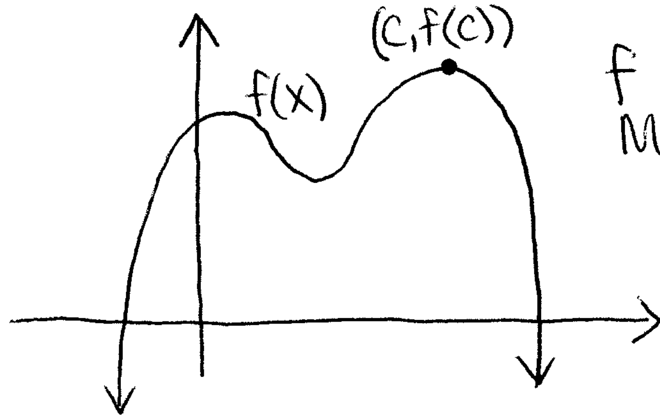
<u>Outline</u>
- Maxima & Minima
- take a quiz

<u>Announcements</u>
- EXAM II October 29 (Ch. 3)
- Scary Math TBA

Maxima & Minima (Chapter 4)

definition: If $f(x)$ is a function, then we say f has an absolute/global MAXIMUM at $x=c$ if $f(x) \le f(c)$ for any x in the domain of f.
→ $f(c)$ is called the <u>MAXIMUM value</u>.

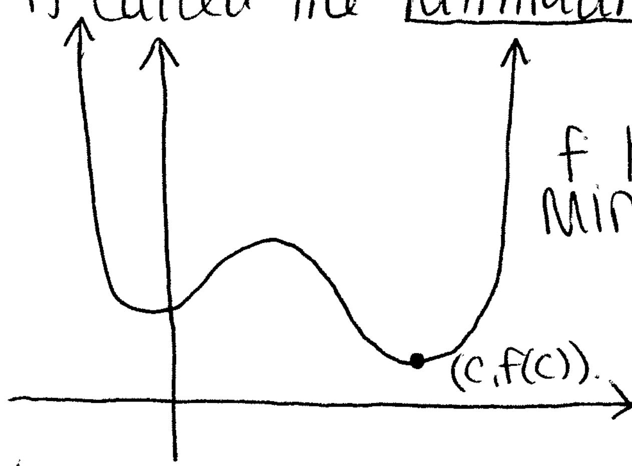
In picture:



f has an absolute MAXIMUM at c.

definition: f has an absolute MINIMUM at c if $f(x) \ge f(c)$ for any x in the domain of f.
→ $f(c)$ is called the <u>MINIMUM value</u>.

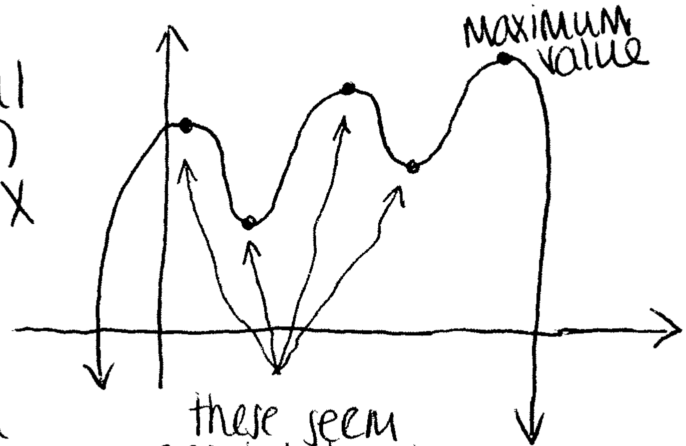
In picture:



f has a global MINIMUM at c.

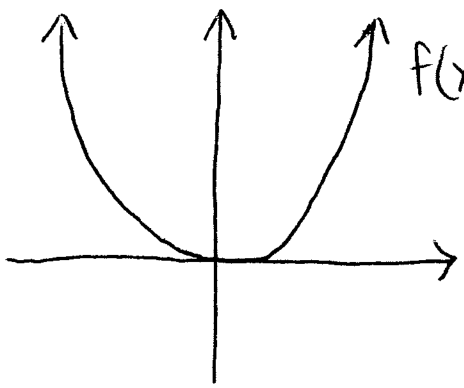
* TERMINOLOGY:
The Max and Min values are sometimes called <u>extrema</u>.
Question: find the extrema of f (Max & Min values).

Definition: f has a local maximum at $x=c$ if $f(x)$ and $f(c)$ for values of x that are near to c. (on both sides of c).



these seem special though they aren't absolute max's/min's.

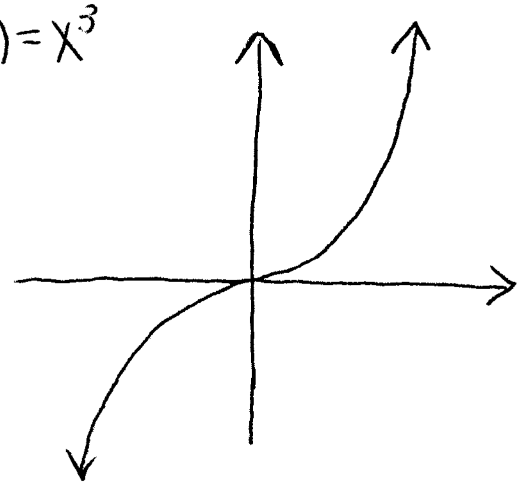
f has a local minima at c if $f(c) \leq f(x)$ for all the x-values near c on both sides.



$f(x)=x^2$

f has a local and global min at $x=0$. the minimum value is $f(0)=0$.

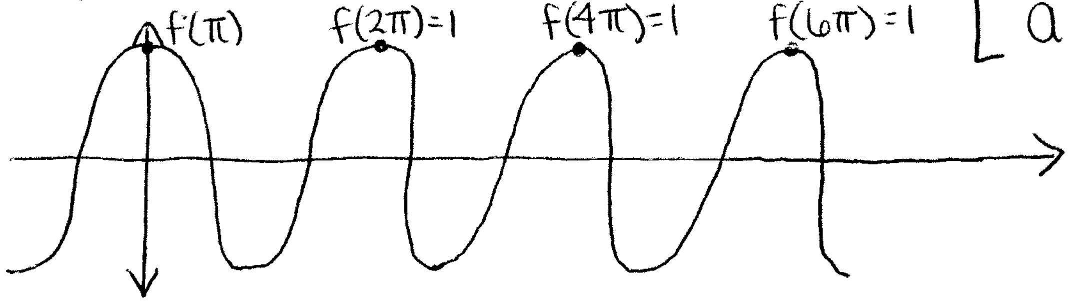
$f(x)=x^3$



- no global maximum or minimum.
- there are no points on the graph that are locally the highest or lowest.

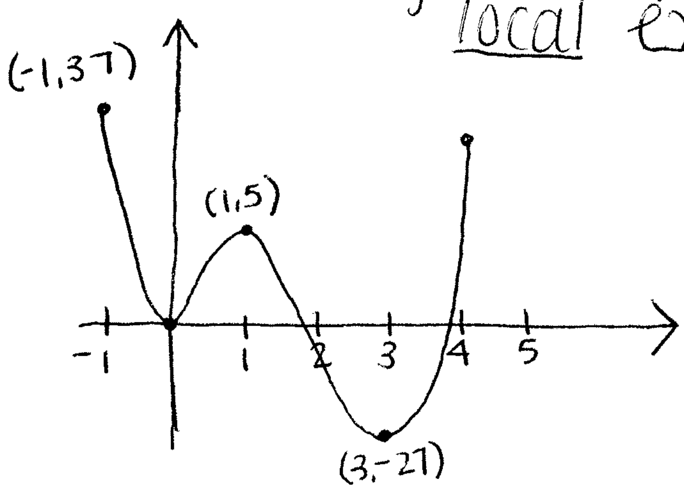
$f(x) = \cos x$

[$f(c) \geq f(x)$ for all x in the domai



Cos(x) has a global Max at every Multiple of 2π ($-2\pi, 0, 2\pi, 4\pi, 6\pi$) because $\cos = 1$ at these numbers and that is greater than or equal to the value cos takes at any other points. Also, cosine has infinitely many global minima.

* Note: These global extrema are also called <u>local</u> <u>extrema</u>.



$f(x) = 3x^4 - 16x^3 + 18x^2$

$-1 \leq x \leq 4$

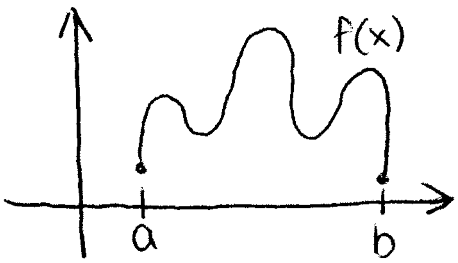
Example: Find all the global & local extrema of f.

- Global Max: f has a global at $x = -1$ and the Max value is 37.
- Global Min: f has a global Min at $x = 3$ and the Min value is -27.
- Local Max: f has a local value at $x = 1$ and the Max value is 5.
- Local Min: global Min & $x = 0$ ← local Min value is 0.

* Note: We don't call $(-1, 37)$ and $(4, 8)$ local extrema because there are not x-values in the domain of f on both sides of -1 and 4.

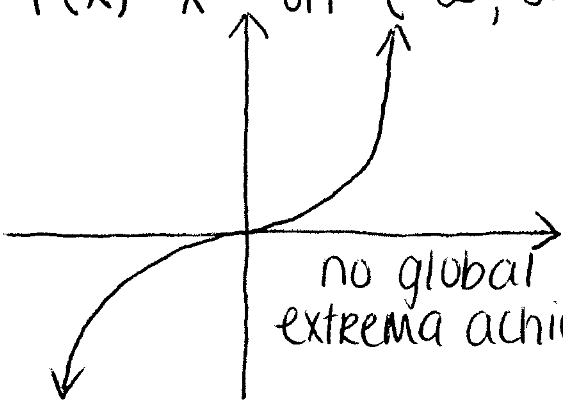
Extreme Value Theorm

If $f(x)$ is continuous on the closed interval $[a,b]$, then f achieves its global maxima and minima $[a,b]$.



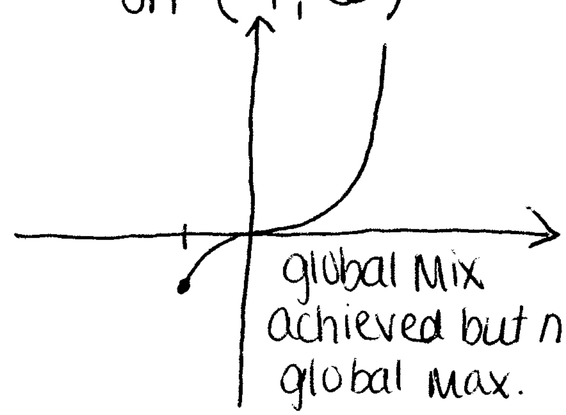
This theorm says: "no way to draw the graph from $x=a$ to $x=b$ and not pass thru the highest and lowest points.

$f(x) = x^3$ on $(-\infty, \infty)$



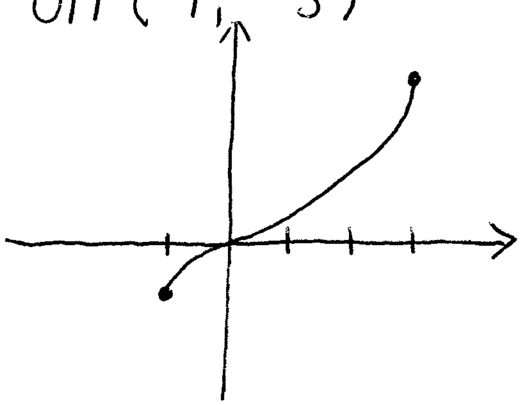
no global extrema achieved

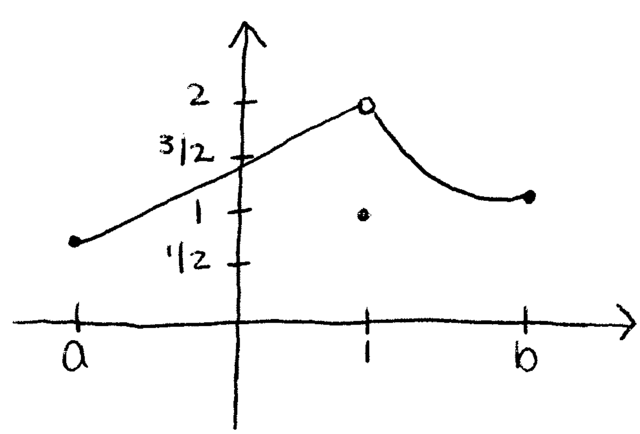
on $(-1, \infty)$



global Min achieved but no global max.

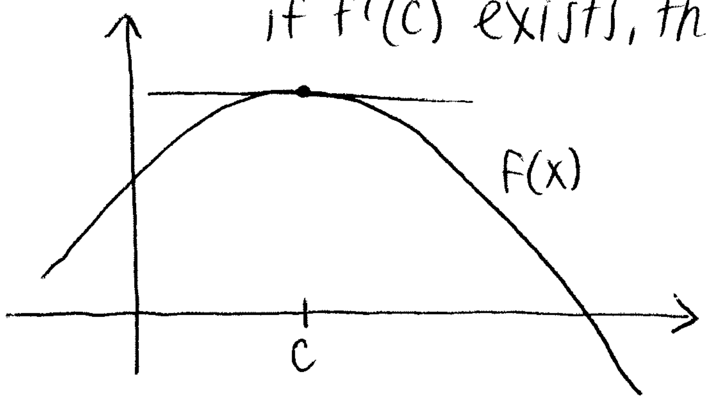
on $(-1, -3)$



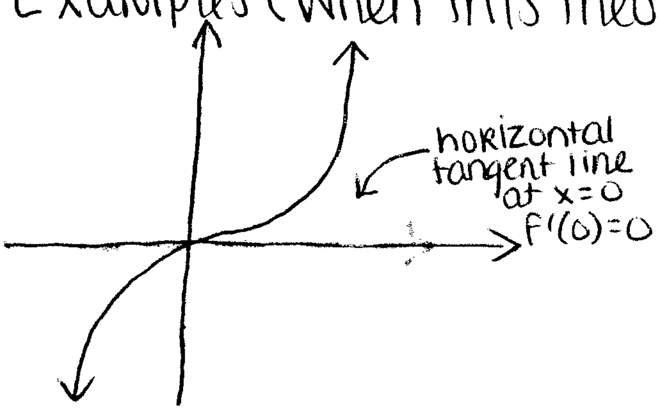


not continuous. no max value.

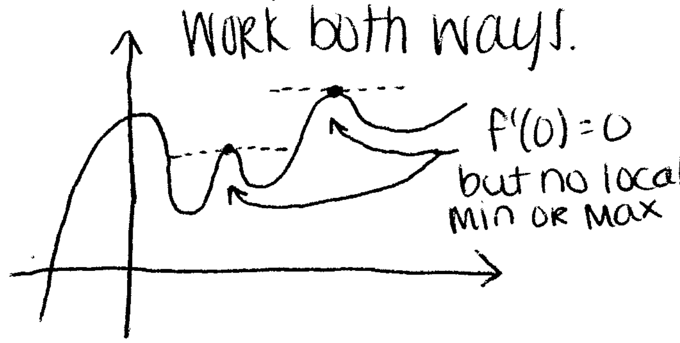
Theorm: if f has a local Max or Min at c and if $f'(c)$ exists, then $f'(c)=0$.



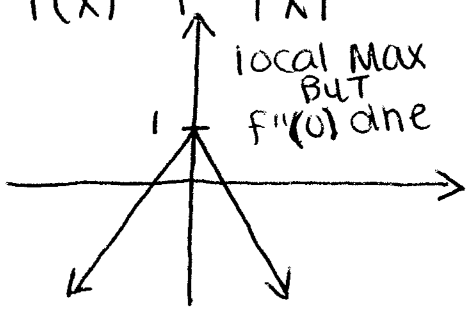
Examples (when this theorm isn't true):



Point: This theorm doesn't work both ways.



$f(x) = 1 - |x|$

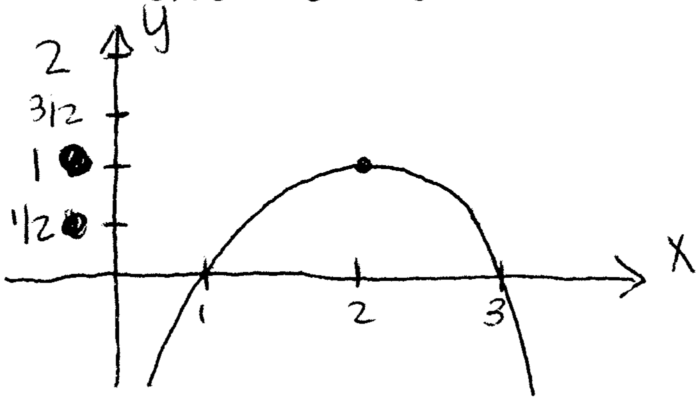


definition: we call c a critical point/number for f is either $f''(c)=0$ or $f'(c)$ dne.

[Strategy for locating extrema of f starts by locating the critical points of f.]

Example from textbook:

11. (a) Sketch the graph of a function that has a local maximum at 2 and is differentiable at 2.



(b) Sketch the graph of a function that has a local maximum at 2 and is continuous but not differentiable at 2.