10/21/09

Outline:

~ finish linear approximations and differentials

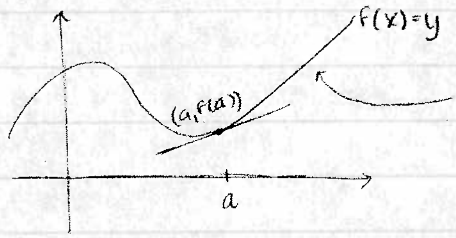
~ maxima/minima

Quiz tomorrow:

~ Baby physics applications of derivatives

~ linear approximations

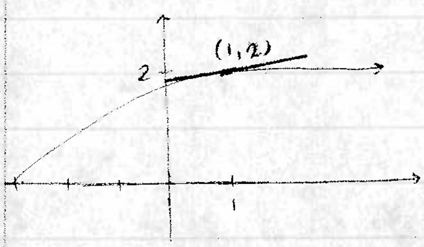
Linear Approximations:



$y = L(x) = f(a) + f'(a)(x-a)$

(linear approx. of f at x=a)

~ ex: $f(x) = \sqrt{x+3}$



$f(1) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}} \cdot 1$

$= \frac{1}{2\sqrt{x+3}}$

$f'(x) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

so: $L(x) = 2 + \frac{1}{4}(x-1)$

~ we can use this to estimate the value $\sqrt{4.05}$, how?

$f(1.05) = \sqrt{4.05}$

$f(1.05) = \sqrt{1.05+3}$

$= \sqrt{4.05} = 2.01246\ldots$

$L(1.05) = 2 + \frac{1}{4}(1.05-1)$

$= 2 + \frac{1}{4}(0.05)$

$= 2 + 0.0125$

$= 2.0125$

$\sqrt{5} = 2.23606797...$

$f(2) = \sqrt{5}$ $\quad L(2) = 2 + \frac{1}{4}(2-1) = 2.25$

$\sqrt{6} = 2.44948974...$

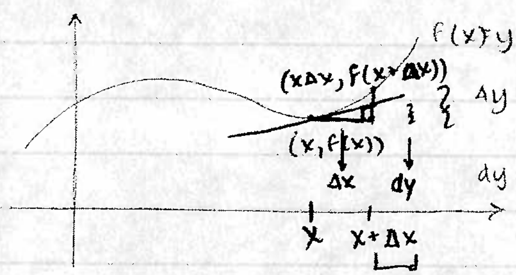
$f(3) = \sqrt{6}$ $\quad L(3) = 2 + \frac{1}{4}(3-1) = 2.5$

Differentials:

~ if $y = f(x)$ then the differential of f, which is written "dy" is: $dy = f'(x)\,dx$

· x & "dx" are input for dy

~ idea by picture * also in text.



} Δy = change in f from x to $x + \Delta x$ = $f(x+\Delta x) - f(x)$

dy = change in $L(x)$ from x to $x + \Delta x$

· tiny amt. added to x

- POINT: if we set $dx = \Delta x$ in the formula for dy, then dy = the height change on the tangent line at $(x, f(x))$ from x to $x + x\Delta$

~ ex: compare Δy & dy for $y = f(x) = x^3 + x^2 - 2x + 1$ when x changes from ⓐ 2 to 2.05 & ⓑ 2 to 2.01. (this is estimating how good the linear approx. is)

$dy = f'(x)\,dx = (3x^2 + 2x - 2)\,dx$

at $x = 2$, $dy = f'(2)\,dx = 14\,dx$

ⓐ when $\Delta x = .05$ then $dy = 14(.05)$ $\qquad \Delta y = f(2.05) - f(2)$

$dy = .7$ $\qquad = .717625$

SO: Δy & dy are close

→

~ What this means: If we choose to use $L(x)$ at $x=2$ for $f(x)$, then near 2 (say 2.05) the value of $L(x)$ differs from $f(x)$ by only a tiny amount.

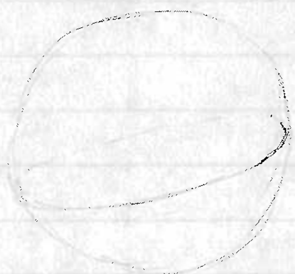
$|dy - \Delta y| = 0.017625.$

ⓑ if $\Delta x = 0.01$ then $dy = 14(0.01)$ $\qquad$ $\Delta y = f(2.01) - f(2)$

$dy = .14$ $\qquad$ $= 0.140701$

So: again, $\Delta y \sim dy$

~ ex:



Sphere:

~ you measure its radius at 21 cm

~ Based on your equipment, there's a possible error in the measurement of 0.05 cm

Q: What is the maximum error you would incur in volume if you use 21 cm as the radius?

~ use differentials to answer this:

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$ $\leftarrow$ (diff'l of V)

Set $dr = 0.05$

$r = 21$

$dV = 4\pi(21)^2(0.05)$

$= 277 \text{ cm}^3$

A: 277 cm^3 is the maximum error in volume you can get, using radius 21 cm.

~ Terminology:

- "relative error" = $\frac{dV}{V} = \frac{277}{\frac{4}{3}\pi(21)^3} = 0.007 = .7\%$

Problem from Text:

§3.9

pg. 193

1 $L(x) = f(a) + f'(a)(x-a)$

$= 4 + (-10)(x+1)$

$= 4 - 10x - 10$

$L(x) = -10x - 6$

$f(x) = x^4 + 3x^2$

$f'(x) = 4x^3 + 6x$

$a = -1$