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10/15 Notes

### Outline

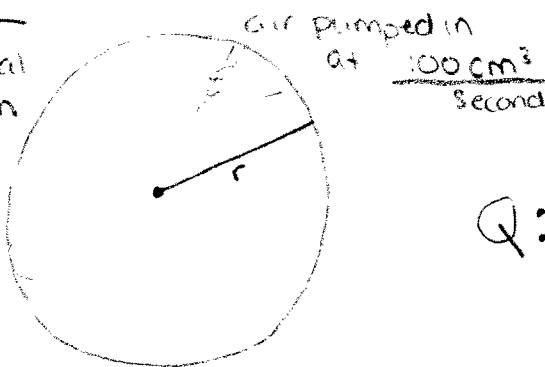
- related rates

Hw due Wed  $\rightarrow$  10/13, 14, 15

## Related Rates (using derivatives)

Ex

Spherical  
balloon



Q: What's the rate (cm/s) in which the radius is expanding? when the diameter is 50 cm.

Related rates:

- rate at which radius is growing
- rate at which volume is growing

Call them  $r(t)$  &  $V(t)$

$\downarrow$   
radius at  
time  $t$  seconds

$\downarrow$   
volume at time  
 $t$  seconds

rates:  $\frac{dr}{dt}$  &  $\frac{dV}{dt}$

$$\left. \frac{dr}{dt} \right|_{r=25} = ?$$

$$100 = \frac{dv}{dt}$$

$$V = \frac{4}{3}\pi r^3$$

chain  
rule  
↓

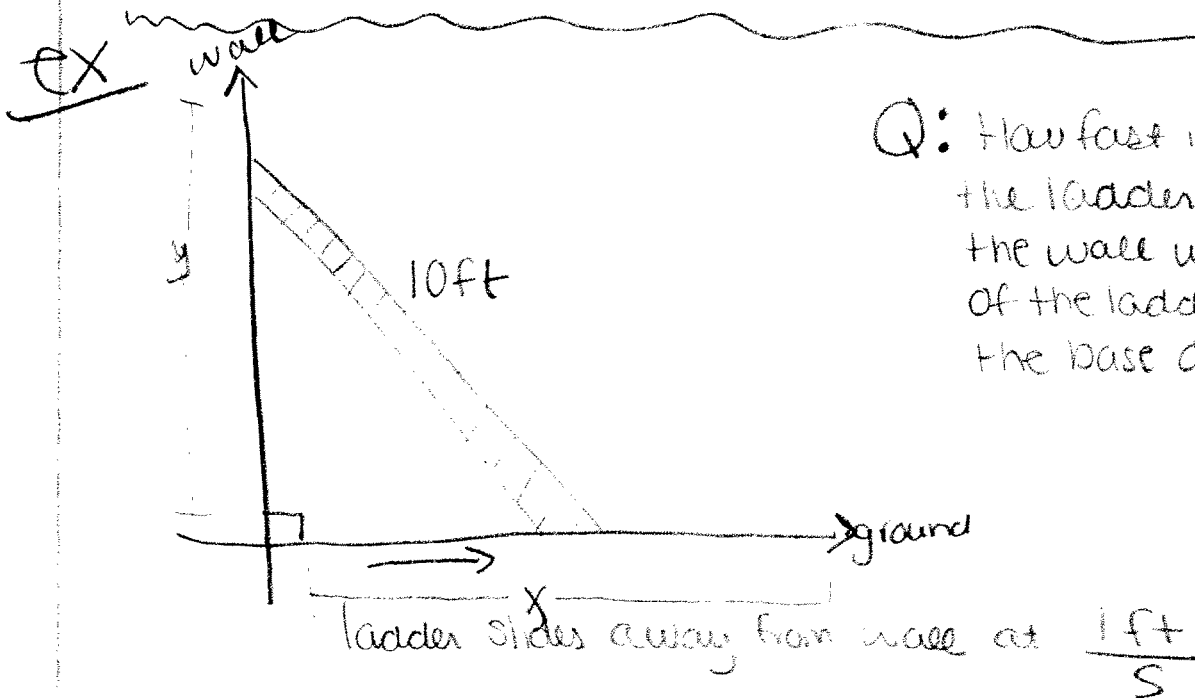
$$\frac{d}{dt} \left( V = \frac{4}{3}\pi r^3 \right) : \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

At the time we care about,  $\frac{dv}{dt} = 100$  &  $r = 25$  cm.

$$100 = 4\pi 25^2 \frac{dr}{dt}$$

$$\frac{1}{25\pi} = \frac{dr}{dt}$$

cm/s



x & y are changing and so the rates are!

$$\frac{dx}{dt} \quad \& \quad \frac{dy}{dt}$$

Q:  $\frac{dy}{dt} \Big|_{x=6} = ?$

How do x & y relate to each other?

$$x^2 + y^2 = 100$$

$$\downarrow \frac{d}{dt}$$

$$2x + 2y \frac{dy}{dt} = 0$$

at the time  
 $x = 6$

$$y = 8$$

$$6^2 + 8^2 = 100$$

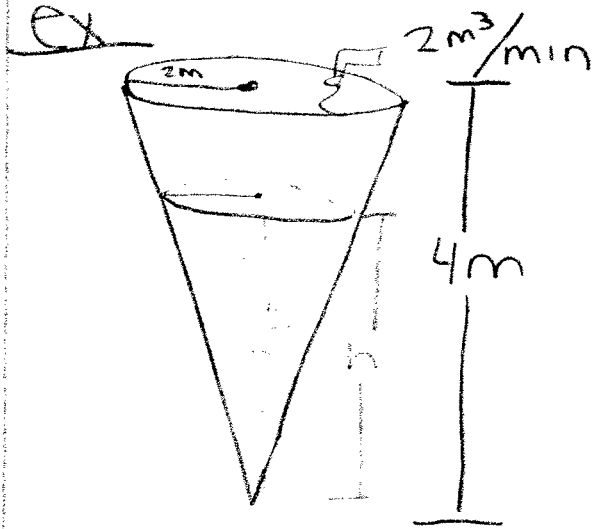
$$\frac{dx}{dt} = 1 \text{ (ft/s)}$$

$$2 \cdot 6 \cdot 1 + 2 \cdot 8 \frac{dy}{dt} = 0$$

$$\boxed{\frac{dy}{dt} = \frac{-12}{16} \text{ ft/s}}$$

Strategy:

- ① Read, read, READ
- ② Draw picture (if possible)
- ③ Label everything that depends on time
- ④ Express the quantities from ③ in any relationships
- ⑤ Identify the rates that you know & want to know
- ⑥ Take derivatives
- ⑦ Substitute the info you know into result of ⑥.



Q: How fast is the water level rising when the water is 3 m deep?

Hint:

$$Vol(\text{cone}) = \frac{1}{3} \pi r^2 h$$

changing:

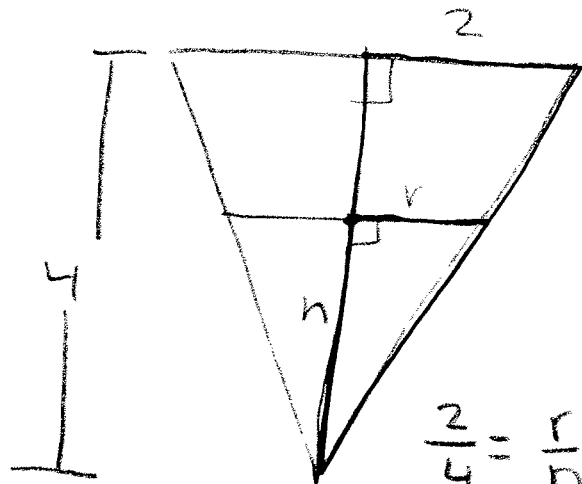
$h, V, r$

$$V = \frac{\pi}{3} r^2 h$$

$$h = 2r$$

$$r = \frac{h}{2}$$

from the side:



$$\frac{2}{4} = \frac{r}{h}$$



$$h = 2r$$

$$Q: \left. \frac{dh}{dt} \right|_{h=3} = ?$$

$$\frac{d}{dt} \left( V = \frac{\pi}{3} r^2 h \right) \rightarrow \frac{dV}{dt} = \frac{d}{dt} \left( \frac{\pi}{3} \left( \frac{r}{2} \right)^2 h \right)$$

$$\frac{dV}{dt} = \frac{d}{dt} \left( \frac{\pi h^3}{12} \right)$$

↓

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

$$= \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$h = 3$$
$$\frac{dV}{dt} = 2 \left( \frac{m^3}{min} \right)$$

$$2 = \frac{\pi}{4} \cdot 9 \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{8\pi}{9} \text{ m/min}}$$

Problem from textbook  
Sec 3.8 #19



$$\frac{dh}{dt} = 1 \text{ cm/min}$$

$$\frac{da}{dt} = 2 \text{ cm}^2/\text{min}$$

$$\frac{db}{dt} = ?$$

When  $h=10$   
 $a=100$

$$100 = \frac{1}{2} b(10)$$
$$b=20$$

$$A = \frac{1}{2} bh$$

$$\frac{da}{dt} = \frac{1}{2} \left( \frac{db}{dt}(h) + \frac{dh}{dt}(b) \right)$$

$$2 = \frac{1}{2} \left( \frac{db}{dt}(10) + 20 \right)$$

$$2 = 5 \frac{db}{dt} + 10$$

$$-12 = 5 \frac{db}{dt}$$

$$\boxed{-\frac{12}{5} = \frac{db}{dt}}$$