Allie Hunt It feels like Monday.

Outline
- Chain Rule
- ???

Announcements
Exam II Thursday 10/29
Quiz Thurs. HW next Wed. (what gets assigned this week

October 13th

Chain Rule

$\frac{d}{dx}(x^2)$ $\frac{d}{dy}(y^3-1)$ - Don't apply chain rule

what's different about these?

$\frac{d}{dx}(\sin(x^2))$ - Do apply chain rule

Practice w/ identifying the inside / outside of composite functi
one function in another func

(1) $(x^4+1)^3 = F(x)$

inside $g(x) = x^4+1$

$f(u) = u^3$

$f(g(x)) = f(x^4+1) = (x^4+1)^3$

Say "F is the composition of f and g"

(2) $\sin^2 x = F(x)$

inside $g(x) = \sin x$

outside $f(u) = u^2$

notation

$f(g(x)) = f(\sin x) = (\sin x)^2 = \sin^2 x$

(3) $\sin(x^2) = F(x)$

inside $g(x) = x^2$

outside $f(u) = \sin(u)$

$f(g(x)) = \sin(x^2)$

④ $F(x) = \frac{1}{x^2+1}$

inside = $g(x) = x^2 + 1$

outside = $f(u) = \frac{1}{u}$

$f(g(x)) = f(x^2+1) = \frac{1}{x^2+1} = F(x)$

⑤ $F(x) = \frac{1}{\sin(\cos x)}$

Can we write $f(u) = \frac{1}{\sin u}$ as a composition of 2 functions?

$g(x) = \cos x$

$f(u) = \frac{1}{\sin u}$

$f(g(x)) = f(\cos x) = \frac{1}{\sin(\cos x)}$

inside $h(u) = \sin u$

outside $k(w) = \frac{1}{w}$

$\rightarrow k(h(u)) = k(\sin u) = \frac{1}{\sin u}$

This means $F(x) = \frac{1}{\sin(\cos x)}$ can be written as the composition of three functions.

inside 1 $g(x) = \cos x$

inside 2 $f(u) = \sin(u)$

outside $k(w) = 1/w$

$k(f(g(x))) = k(f(\cos x)) = k(\sin(\cos x)) = \frac{1}{\sin(\cos x)}$

⑥ $\tan(\pi x) = F(x)$

inside $g(x) = \pi x$

outside $f(u) = \tan(u)$

$f(g(x)) = f(\pi x) = \tan(\pi x)$

(7) $\sqrt{\sin^2(4x)}$

in two ways:

as composition of 2 functions & composition of 3 functions

inside $g(x) = \sin(4x)$ inside 1 $g(x) = 4x$

outside $f(u) = \sqrt{u^2} = u$ inside 2 $f(u) = \sin^2(w)$

out. $k(w) = \sqrt{w}$

$k(f(g(x))) = \sqrt{\sin^2(4x)}$

from the Quiz

$$\frac{\frac{2}{x^3} + \sqrt{x}}{x^2+1} \longrightarrow \sqrt{x^3} = x^{3/2}$$

<u>Chain Rule</u>: If $F(x) = f(g(x))$ then $F'(x) = f'(g(x)) \cdot (g'(x))$

or (in other words) if $y = f(u)$, $u = g(x)$

then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$F(x) = (x^4 + 1)^3$

$g(x) = x^4 + 1$ so $\frac{dF}{dx} = 3(x^4+1)^2\, 4x^3$

$f(u) = u^3$

$F(x) = \sqrt{x^2+1}$ $\frac{dy}{du} = \frac{1}{2}u^{-1/2}$ $\frac{du}{dx} = (x^2+1) + 2$

inside $u = g(x) = x^2+1$

outside $y = f(u) = \sqrt{u}$

so $\frac{dy}{dx} = \frac{1}{2}u^{-\frac{1}{2}}(2x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

fyi $\longrightarrow$ $= \frac{x}{\sqrt{x^2+1}}$

but you don't have to do it.

Rule: If f is any function

$$\frac{d}{dx}\left[f(x^n)\right] = f'(x^n)\cdot nx^{n-1}$$

$$g(t) = \left(\frac{t-2}{t^2+6}\right)^{19}$$

inside $h(t) = t-2/(t^2+6)$

outside $f(u) = u^{19}$

$h'(t) = 1(t^2+6)-(t-2)2t/(t^2+6)^2$

$f'(u) = 19u^{18}$

so $g'(t) = f'(h(t))\cdot h'(t)$

$$= 19\left(\frac{t-2}{t^2+6}\right)^{18}\left[\frac{t^2+6-(t-2)(2t)}{(t^2+6)^2}\right.$$

$F(x) = \sin(\cos(\tan(x)))$

inside 1 $g(x) = \tan x$

inside 2 $f(u) = \cos u$

outside $k(w) = \sin w$

the chain rule has to be used twice

$\cos(\tan(x)) = f(g(x))$

↳ $\frac{d}{dx}$

$f' = (-\sin)$

$g' = \sec^2$

so $\frac{d}{dx}(f(g(x))) = [-\sin(\tan x)]\sec^2 x$

$k'(w) = \cos w$

so $F(x) = \underbrace{k}(\underbrace{f(g(x))})$

$F'(w) = k'^{\text{out}}(f(g(x))^{\text{in}})\cdot\frac{d}{dx}[f(g(x))]$

So $\boxed{F'(x) = \cos(\cos(\tan x)) \cdot [-\sin(\tan x)] \cdot \sec^2 x}$

Note: If $F(x) = k(f(g(x)))$ then
$F'(x) = k'(f(g(x))) \cdot f'(g(x)) \cdot g'(x)$

$y = \sqrt{\sec(x^3)}$

inside 1 $g(x) = x^3$

inside 2 $w = \sec(u)$

Outside $y = \sqrt{w}$

Y depends on w which depe[nds] on U which depends on f(x)

$\frac{dy}{dw} \cdot \frac{dw}{du} \cdot \frac{du}{dx}$

$\frac{1}{2} w^{-1/2} \cdot \sec u \cdot \tan u \cdot 3x^2$

$\frac{1}{2}(\sec(x^3))^{-\frac{1}{2}} \cdot (\sec(x^3)\tan(x^3)(3.3)$

Part 1 of hw due next wed. 10/21 will come from 3.5 1-54

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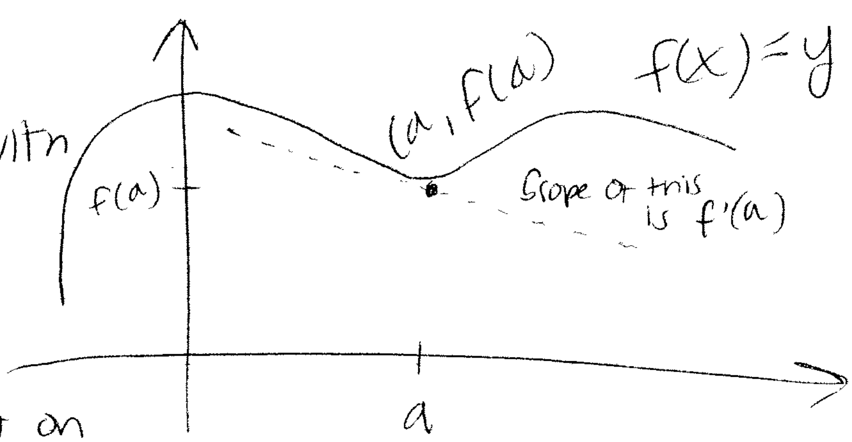
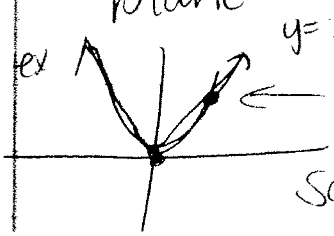
§3.6 Implicit Differentiation

recall:



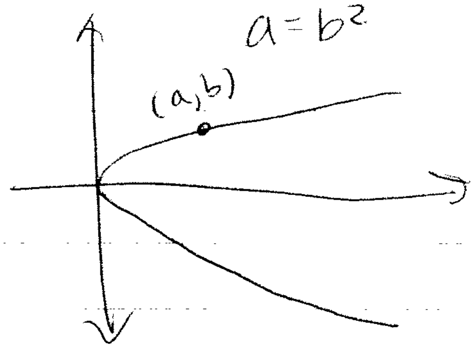
Fact: any equation with x's and y's describes a curve in the plane



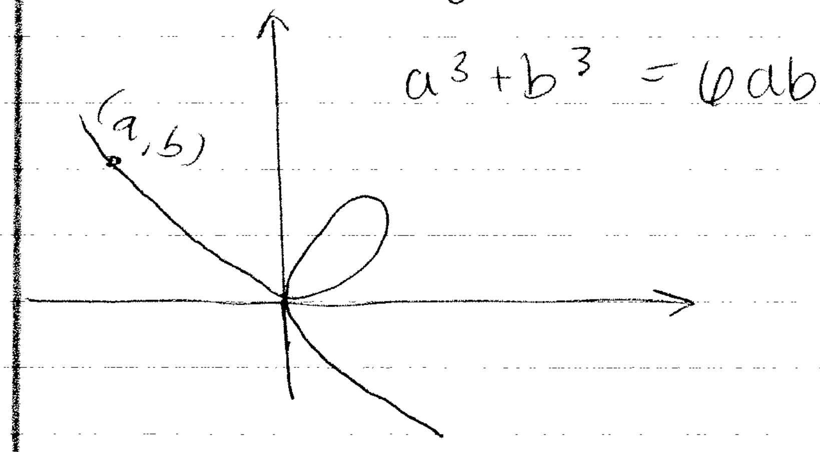
← any point on this curve satisfies

y - coord. = (x coord)^2

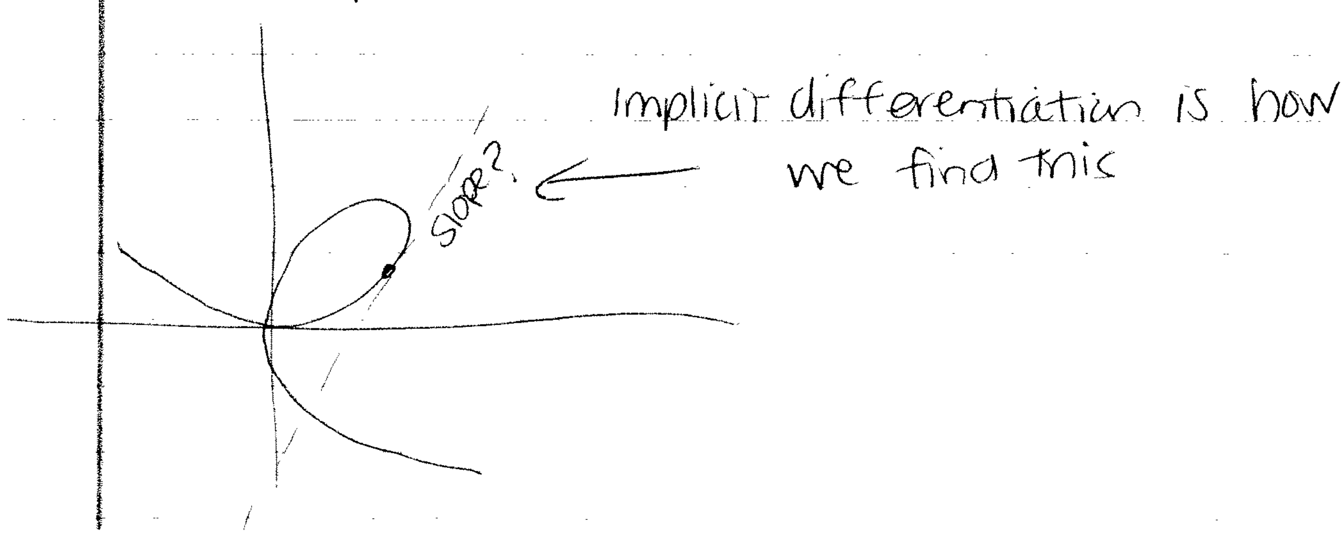
ex. $x = y^2$



$x^3 + y^3 = 6xy$



Some of these curves are <u>not</u> the graph of a function. But we still would like to know what the slope of the tangent line is at a point:



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(1). $y = \sin 4x$
$y' = 4\cos 4x$