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Outline
- Mean Val[illegible] Theorem
- Other stu[illegible]

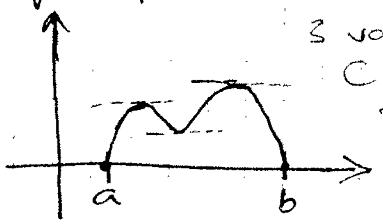
Announce[illegible]
O.H 1|30 – 3 [illegible]
Hw Due Th[illegible]
Tues. Nov. [illegible] 5pm, Scary [illegible] Extra C[illegible]

§ 4.2 MVT (Mean Value Theorem)

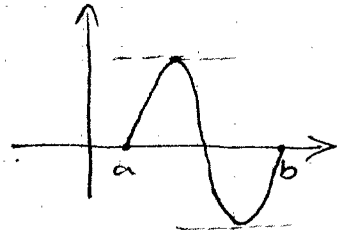
Rolle's Theorem

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), and if $f(a)=0=f(b)$ then there is a C between a & b where $f'(C)=0$

Pics of why true:

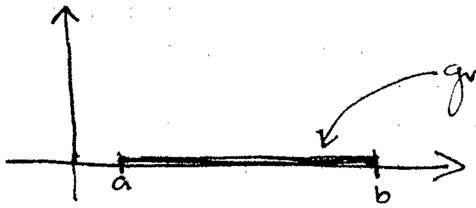


3 values of C where $f'(C)=0$



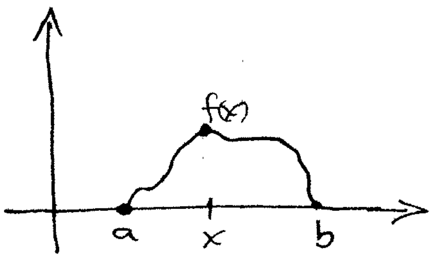
We can actually prove this is true:

Case 1: $f(x)$ = constant (so $f'(x)=0$)



graph. Pick any point $a<c<b$ then $f'(C)=0$

Case 2: There is some x value where $f(x)>f(a)$. ($f(x)$



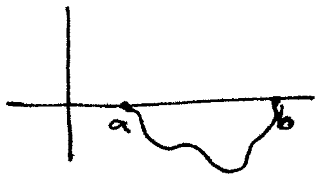
Apply the Extreme Value Theorem to this situation.

recall: EVT says a cts function on $[a,b]$ achieves its global max/min values.

So somewhere between a & b, there has to be a n
value C where f(C) is the maximum height th
reaches.

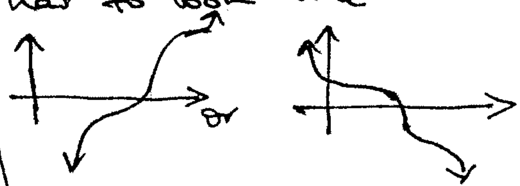
then $f'(C) = 0$
So this is the C we want.

<u>Case 3</u>: same as 2 except that time suppose f looks



<u>Ex</u>: Use this theorem to show that $f(x) = x^3 + x - 1$ h
<u>exactly</u> one real root.

<u>First of all</u>: This polynomial must have <u>at least</u> one
root (b/c an odd degree polynomial
has to look like



or

Show that there is <u>exactly</u> one:

$f(a) = 0 = f(b)$.
(Suppose a & b
are two roots)

If true then f, a, b satisfy Rolle's Theorem
So the conclusion of the theorem tells us that
there must be some value C that is betwe
a & b and where $f'(c) = 0$

$f'(x) = 3x^2 + 1$

so

$f'(C) = 3c^2 + 1 = 0$

$c^2 = -\frac{1}{3}$

$c = \sqrt{-\frac{1}{3}}$ ......?

$f'(x) = 3x^2 + 1 > 0$ always!

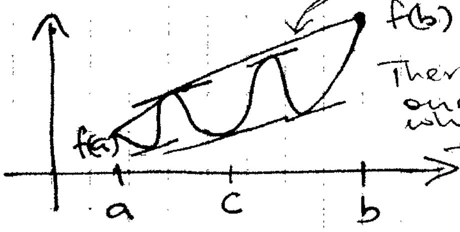
So there can't be 2 roots!
So there is exactly 1 root.

MVT (Mean Value Theorem)

If $f(x)$ is cts on $[a, b]$ and diff'ble on (a, b) then there is some value of c between a & b where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

slope $= \dfrac{f(b) - f(a)}{b - a}$



There is at least one value of c where the slope of the tangent line to the graph at c is equal to the secant line slope from a to b.

Why true: Define a new function

$$h(x) = f(x) - \underbrace{\left[f(a) + \frac{(f(b) - f(a))}{b - a}(x - a) \right]}$$

This is the function whose graph is the secant line.

$h(x)$ satisfies Rolle's Thm:

- $h(x)$ is continuous on $[a, b]$ b/c it is a difference of cts functions.
- $h(x)$ is differentiable on (a, b) for the same reason.
  - Check that $h(a) = 0 = h(b)$

$$h(a) = f(a) - \left[f(a) + \underbrace{\frac{(f(b) - f(a))}{b - a}(a - a)}_{0} \right] = 0$$

$h(b) = 0$

So there is (by Rolle's Theorem), a value $a < c < b$ where $h'(c) = 0$

$$h'(x) = f'(x) - \left(\frac{f(b) - f(a)}{b-a}\right)$$

$$0 = h'(c) = f'(c) - \frac{(f(b) - f(a))}{b-a}$$

$$\frac{f(b) - f(a)}{b-a} = f'(c)$$

<u>Example</u> $f(x) = x^3 - x \qquad a = 0, \ b = 2$

<u>Find</u> the value of c guaranteed by the MVT

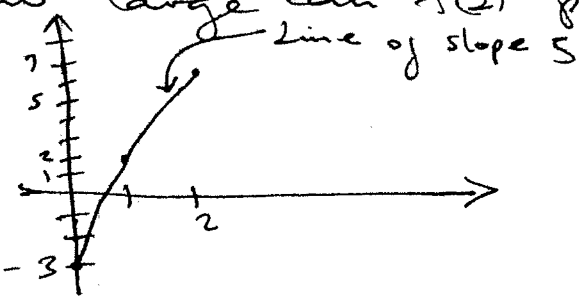
$$\frac{f(2) - f(0)}{2-0} = \frac{6}{2} = 3 \qquad f'(x) = 3x^2 - 1$$

So find the c where $f'(c) = 3$

$$3c^2 - 1 = 3$$
$$3c^2 = 4$$
$$c^2 = 4/3$$
$$c = \pm \frac{2}{\sqrt{3}}$$

So $f'\left(\frac{2}{\sqrt{3}}\right) = \frac{f(2) - f(0)}{2-0} \quad \left(c = \frac{2}{\sqrt{3}}\right)$

<u>Ex</u>: If $f(0) = -3$ and $f'(x) \le 5$ for every x, th
how large can $f(2)$ possibly be?



The mean value theo
says there is a $0 < c$
where

$$\frac{f(2) - f(0)}{2-0} = f'(c)$$

$$\frac{f(2) + 3}{2} = f'(c) \le$$

$\frac{f(2)+3}{2} \le 5 \implies f(2) + 3 \le 10 \quad \boxed{f(2) \le 7}$